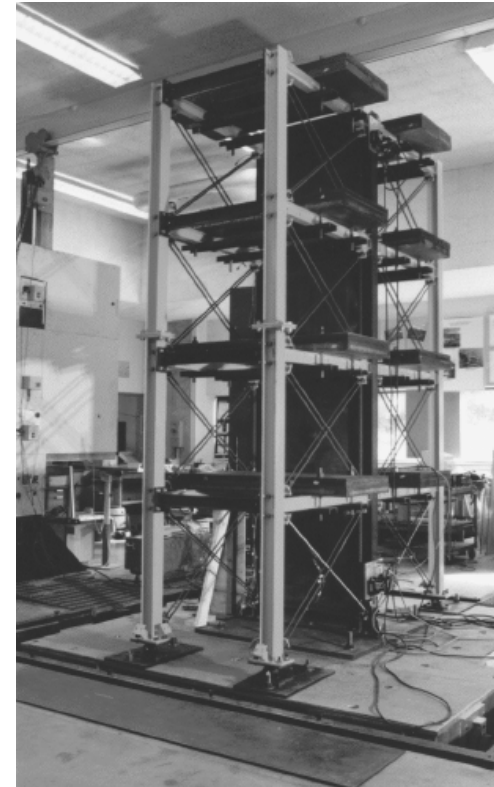
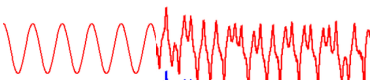


VIBRAÇÕES EM ESTRUTURAS DE AÇO

Zacarias M. Chamberlain Pravia (Ver. 2020)




Zacarias Chamberlain 2020

ASCE
AMERICAN SOCIETY OF CIVIL ENGINEERS



**ESTE MATERIAL É DE PROPRIEDADE DO AUTOR E APENAS É FORNECIDO
COMO FERRAMENTA DE ACOMPANHAMENTO DO CONTEÚDO DO CURSO.**



1 d f d u b v # k d p e h u o l q # 5 3 5 3

CONTEÚDO

1 - Introdução as vibrações, porque é necessário saber dinâmica?

2 - Sistemas de Um grau de liberdade

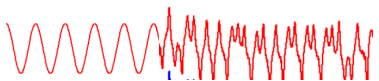
2.1 – Vibrações livres – equação de movimento

2.2 – Vibração livre não amortecida (frequência natural)

2.3 – Vibração livre amortecida

2.4 - Determinação de frequências naturais cálculo manual

2.5 - Determinação de frequências naturais usando programas computacionais



CONTEÚDO

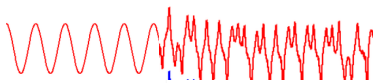
3 – Sistemas de múltiplos graus de liberdade

3.1 – Sistema não amortecido

3.2 - Frequências e modos de vibração

3.3 – Análise Modal de Vibrações livres

3.4 – Análise Modal de vibrações forçadas



CONTEÚDO

3 – Problemas dinâmicos nas estruturas de aço e soluções

3.1 – Vibrações induzidas por equipamentos rotativos

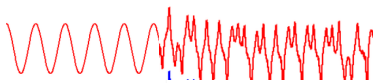
3.2 – Vibrações induzidas pelo vento

3.3 – Vibrações produzidas por movimento humano (ginásios, passarelas, pisos)

3.4 – Vibrações produzidas na construção civil

5.6 - Medições de frequências naturais.

4 – Medição de frequências Naturais (aula prática)



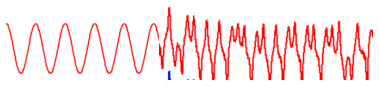
REFERÊNCIAS

[1] BRASIL, R., SILVA, M.A., **Introdução à dinâmica das estruturas para Engenharia Civil**, 2ª edição, Editora Edgar Blucher, 2017.

[2] SORIANO, H. LIMA, **Introdução à dinâmica das estruturas**, ELSEVIEAR Editora Limitada, 2014.

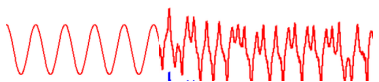
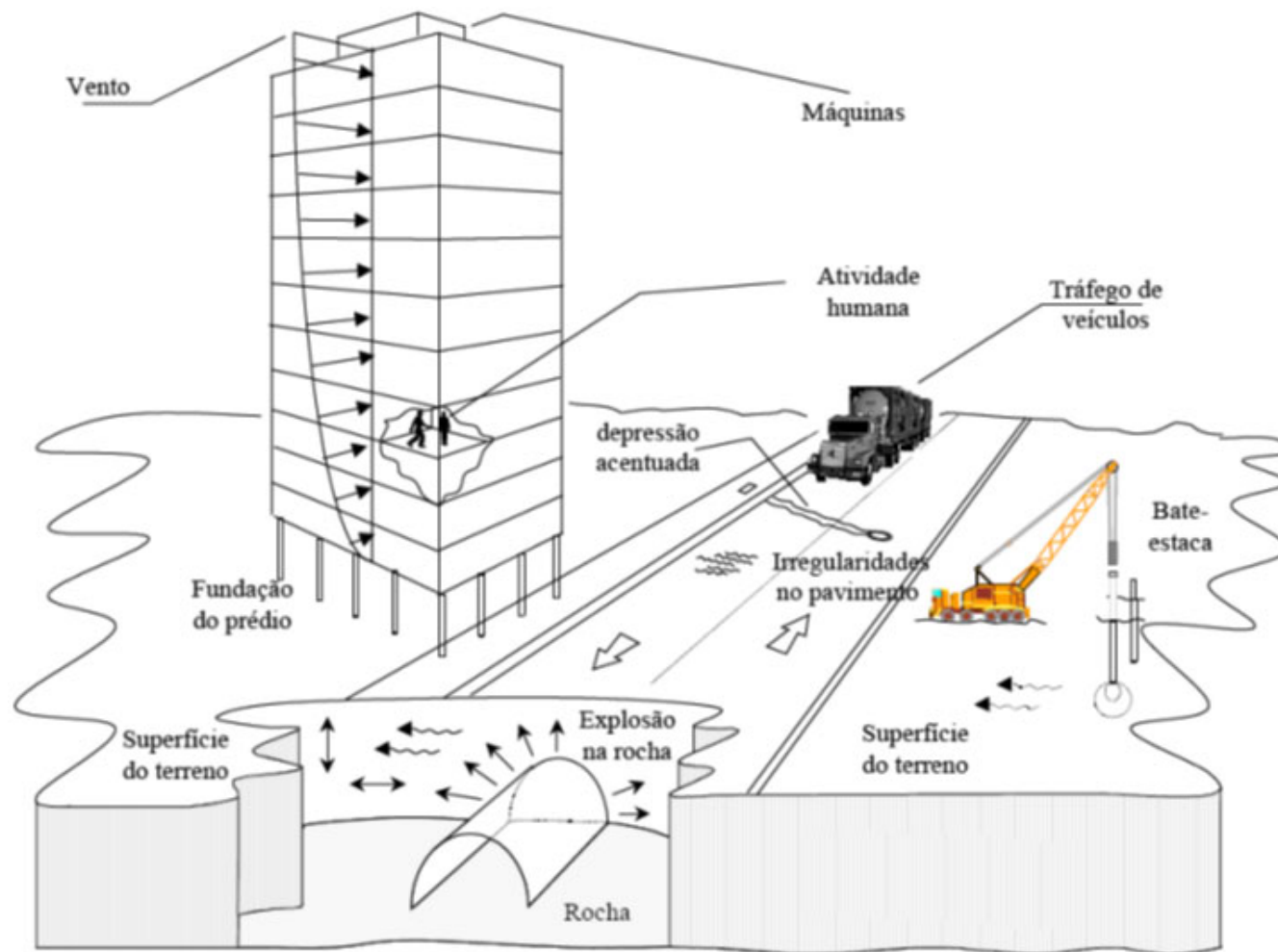
[3] **Vibrações em Pavimentos: Recomendações Técnicas de Projecto**, RFS2-CT-2007-00033

[[4] Murray et al, *Vibrations of Steel-Framed Structural Systems Due to Human Activity*, Second edition, AISC Design Guide, 2016.

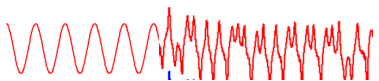
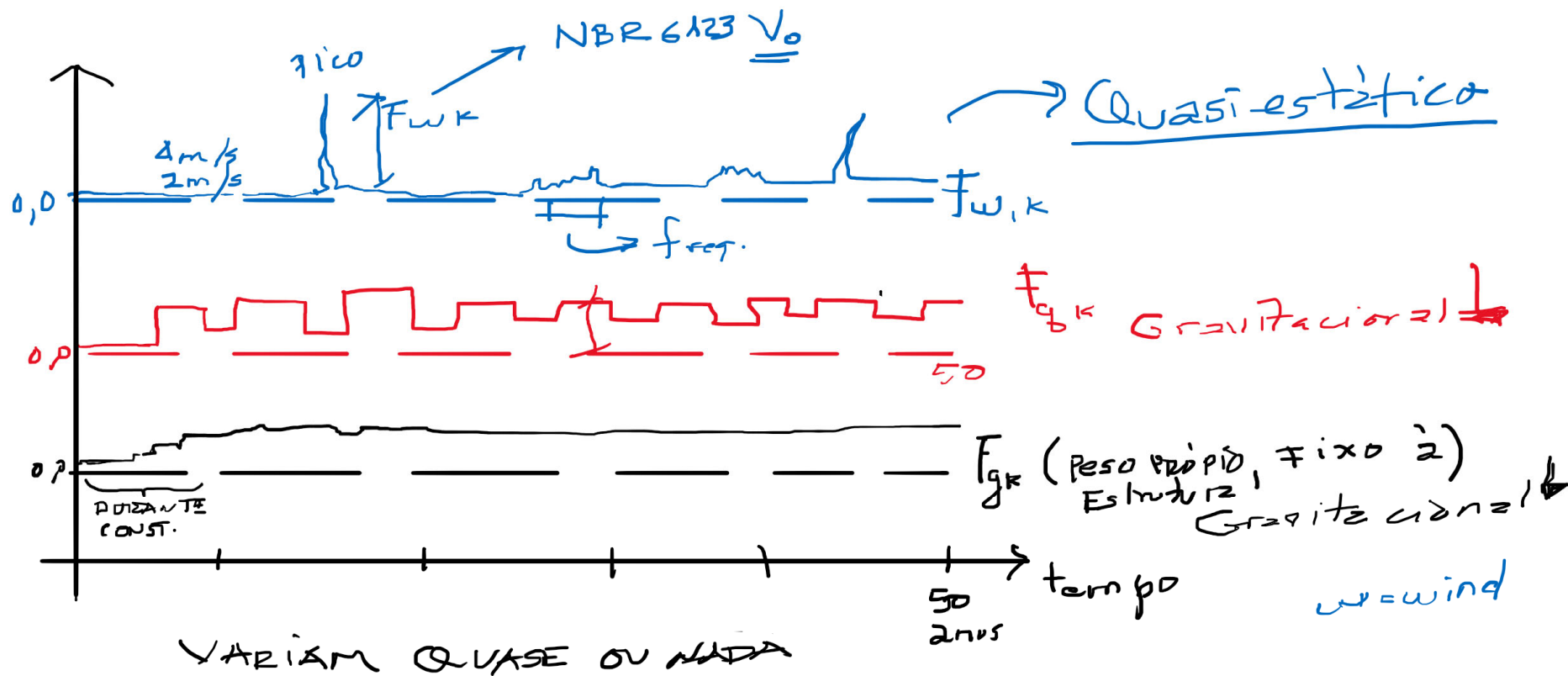


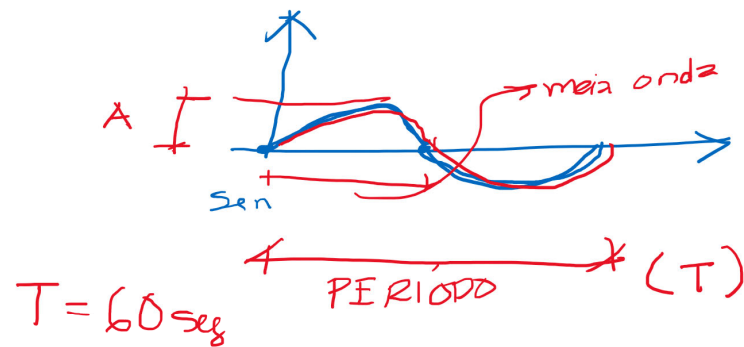
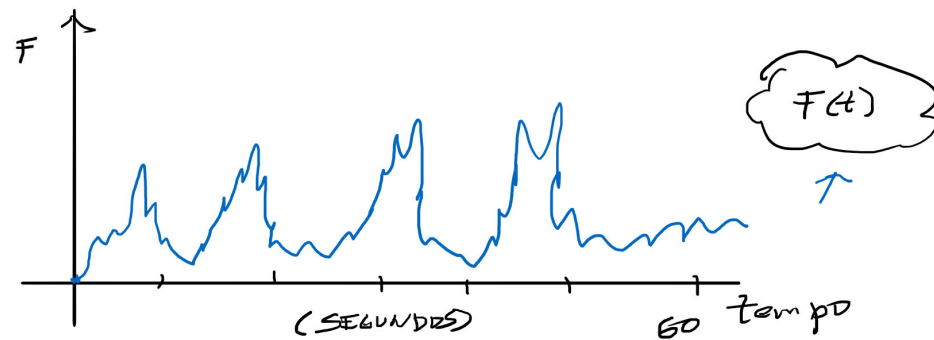
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(1) PORQUÊ VIBRAÇÕES?

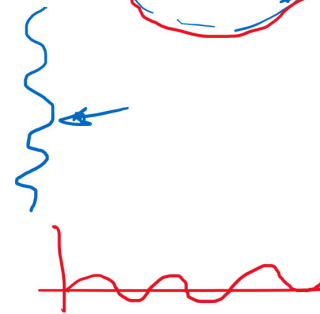
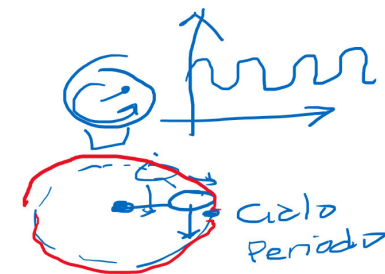


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$$f = \frac{1}{T} = \frac{1 \text{ ciclo}}{60 \text{ seg}} = \frac{1}{60}$$



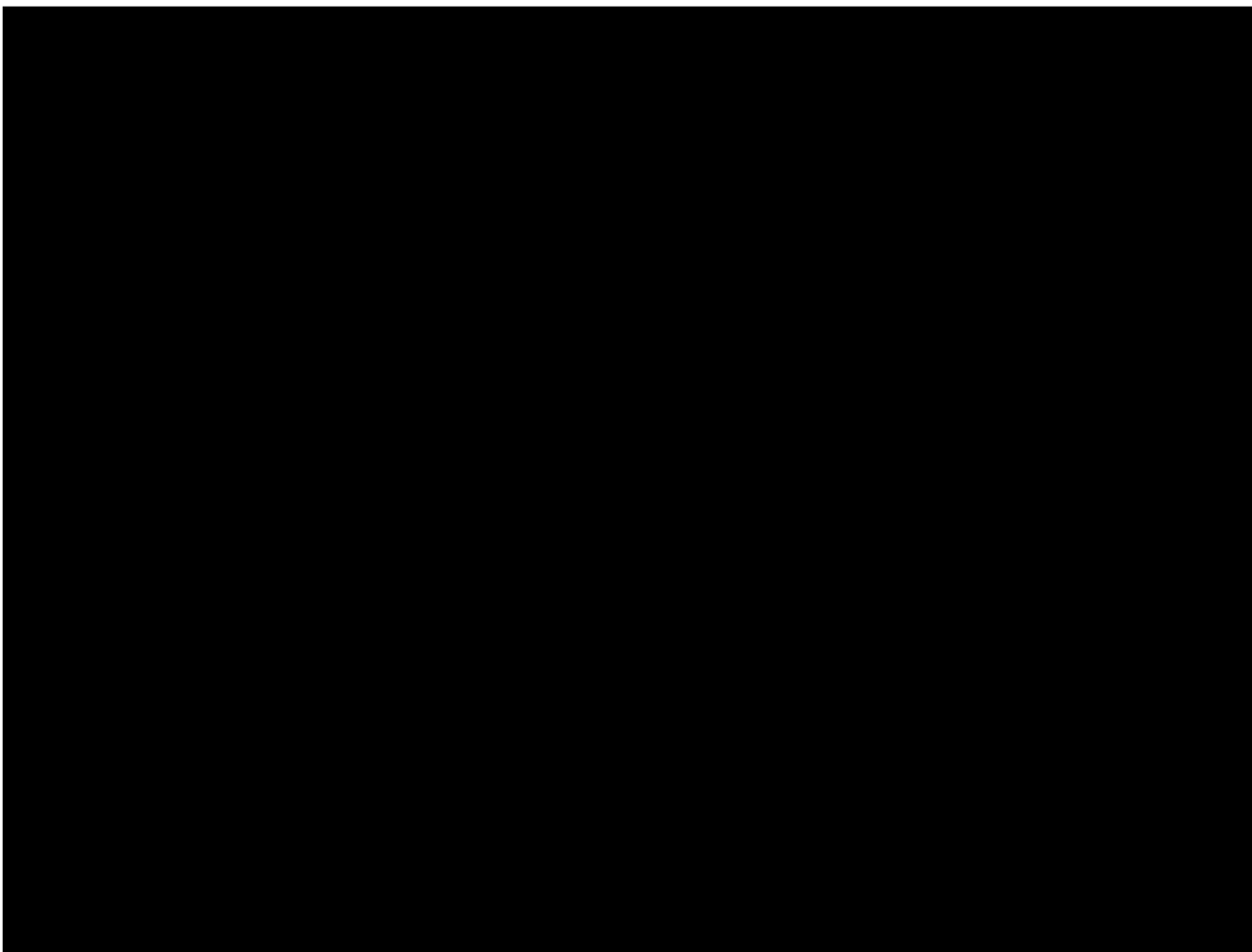
$$\frac{1}{s} = \text{Her}^-$$

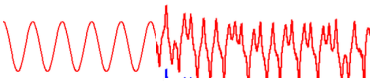


A red, jagged, scribbled line, resembling a stylized signature or a series of connected 'v' shapes, is positioned above the text 'Zacarias Chamberlain 2020'.

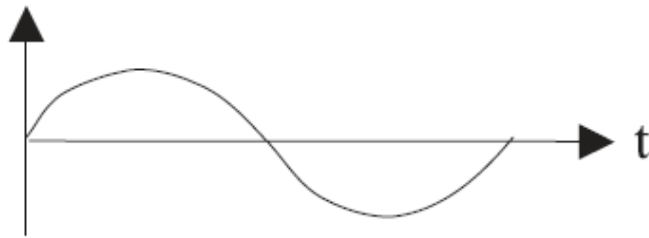
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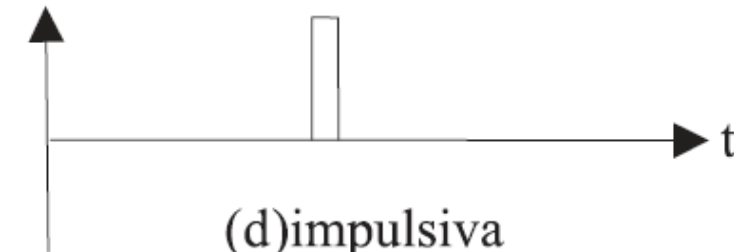
(a) harmônica



(c) transiente

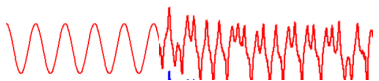


(b) periódica

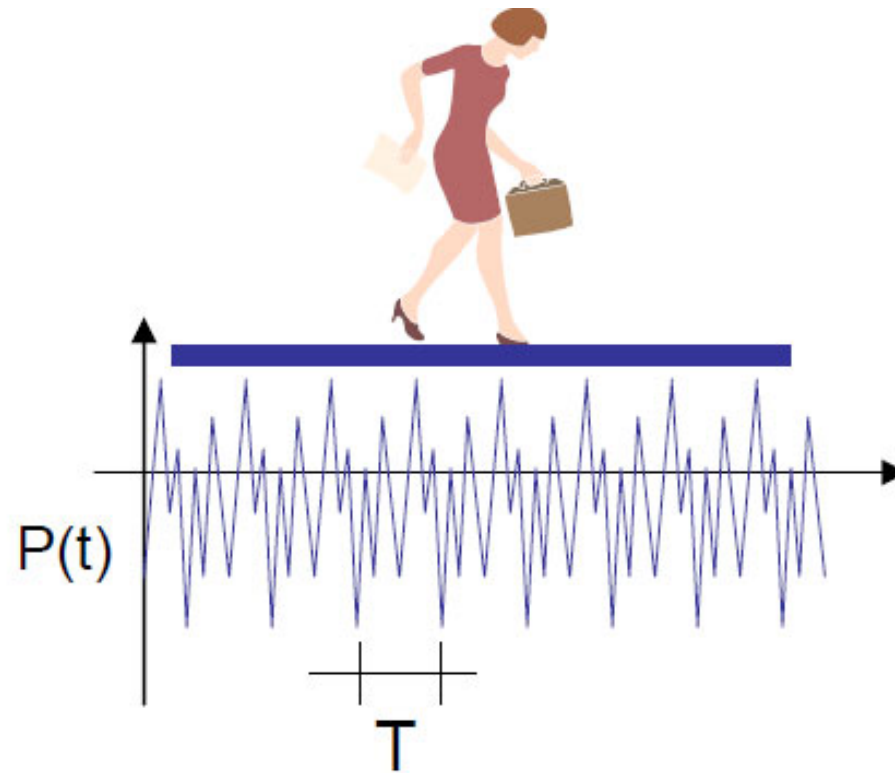


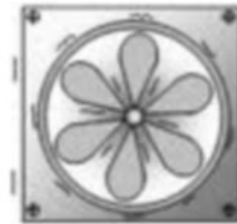
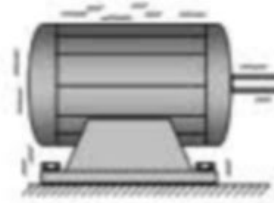
(d) impulsiva

TIPOS DE CARGAS DINÂMICAS



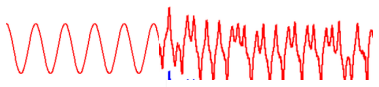
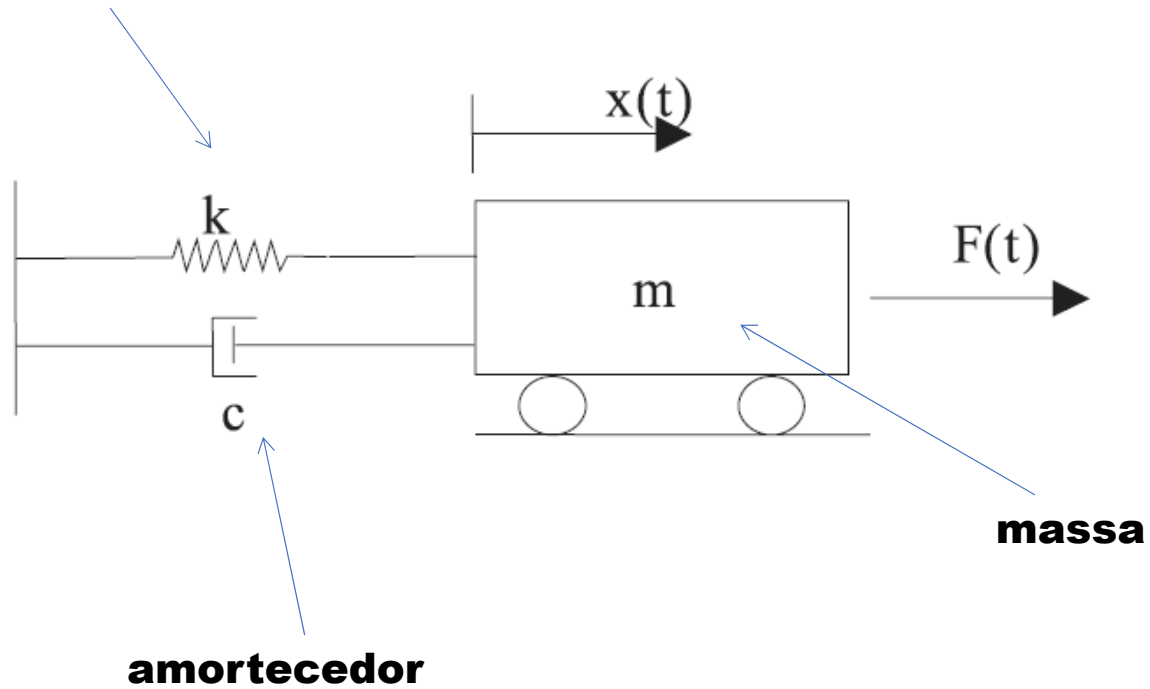
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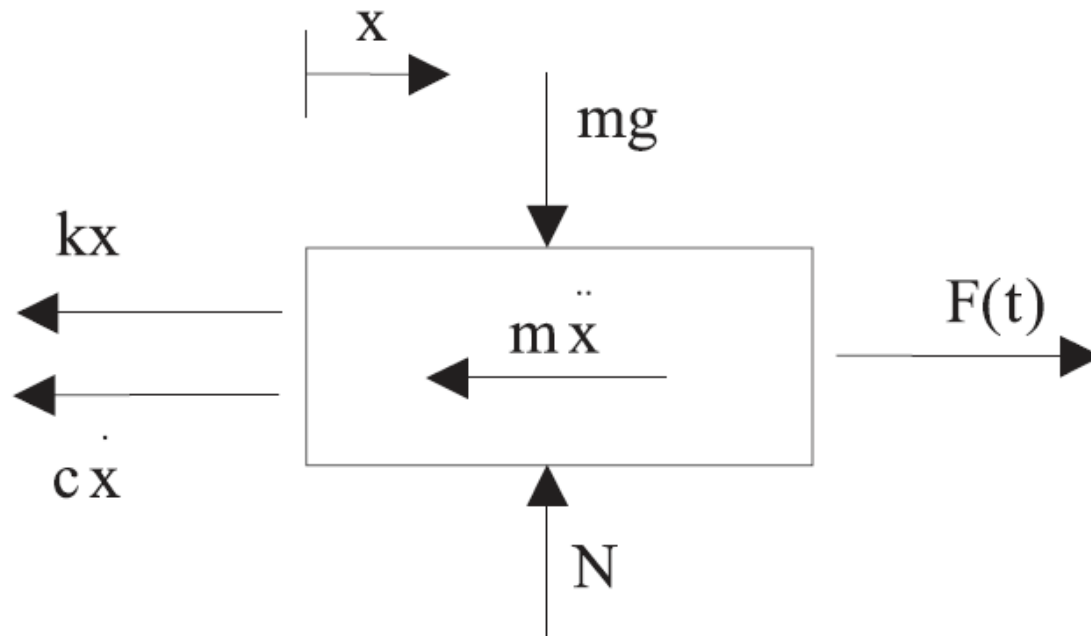
(1) SISTEMAS DE 1 GRAU DE LIBERDADE

mola(rigidez)

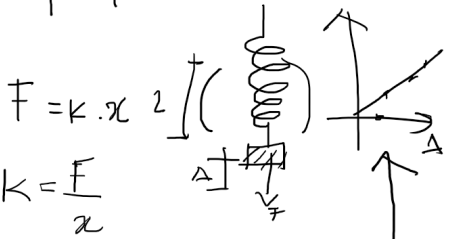
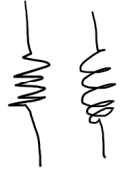
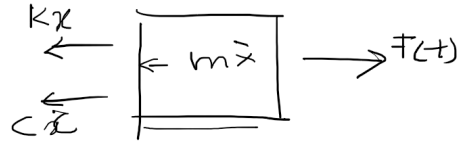


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(1) SISTEMAS DE 1 GRAU DE LIBERDADE



$$m \ddot{x}(t) + c \dot{x}(t) + kx(t) = F(t)$$



$$F = k \cdot x$$

$$k = \frac{F}{x}$$

$$k = \frac{F}{L} = \# L^{-1}$$

$$k = \frac{N}{mm}$$

$$\sigma = \frac{P}{A}$$

$$\sigma = E \cdot \epsilon$$

$$\epsilon = \frac{\delta}{L}$$

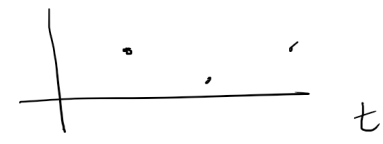
$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

$$\delta = \frac{PL}{EA}$$

$$F = k \cdot x$$

$$k = \frac{EA}{L}$$

$$F = \frac{EA}{L} x$$



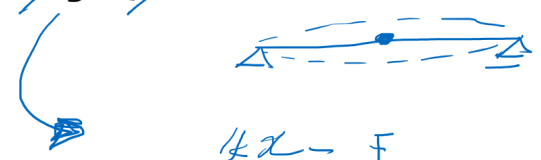
$$x(t)$$

$$\frac{dx(t)}{dt} = v = \dot{x}$$

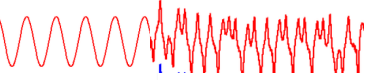
$$\frac{d^2x(t)}{dt^2} = a = \ddot{x}$$

EQ. DINÁMICA

$$m\ddot{x} + c\dot{x} + kx = F(t)$$



EQ. ESTÁTICA



$$m \ddot{x} + c \dot{x} + kx = F(t) \quad \text{Sem amort.}$$

Vibrações Livres

$$m \ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\rightarrow \ddot{x} + p x = 0 \quad \text{Eq. 2 grau}$$

$$x = A \sin(ct)$$

$$\dot{x} = A c \cos(ct)$$

$$\ddot{x} = -A c^2 \sin(ct)$$

$$-A c^2 \sin(ct) + p A \sin(ct) = 0$$

$$-c^2 + p = 0 \quad \therefore c^2 = p$$

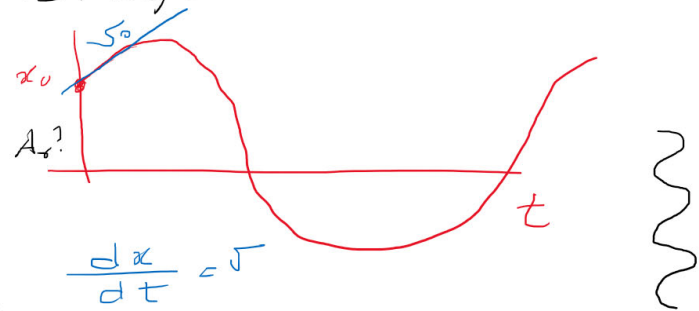
$$c = \sqrt{\frac{k}{m}}$$

ω = freq. angular

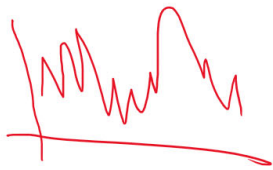
$$\omega = \sqrt{\frac{k}{m}} \quad \leftarrow \text{rigidez}$$

$m = \text{massa}$

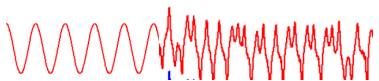
V_0
 x_0

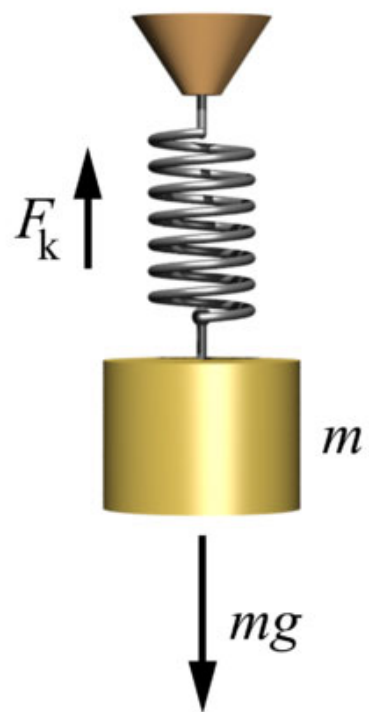
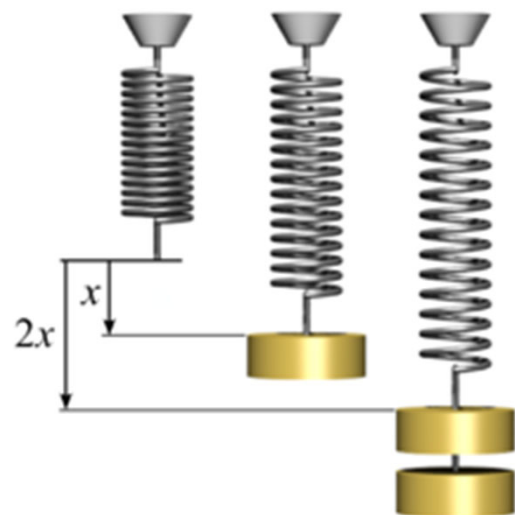


$$p = \frac{k}{m}$$



Fourier
Series





(1) SISTEMAS DE 1 GRAU DE LIBERDADE

VIBRAÇÃO LIVRE

$$\omega_n = \sqrt{\frac{k}{m}}$$

Frequência
Circular

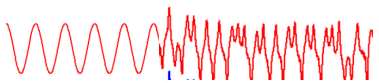
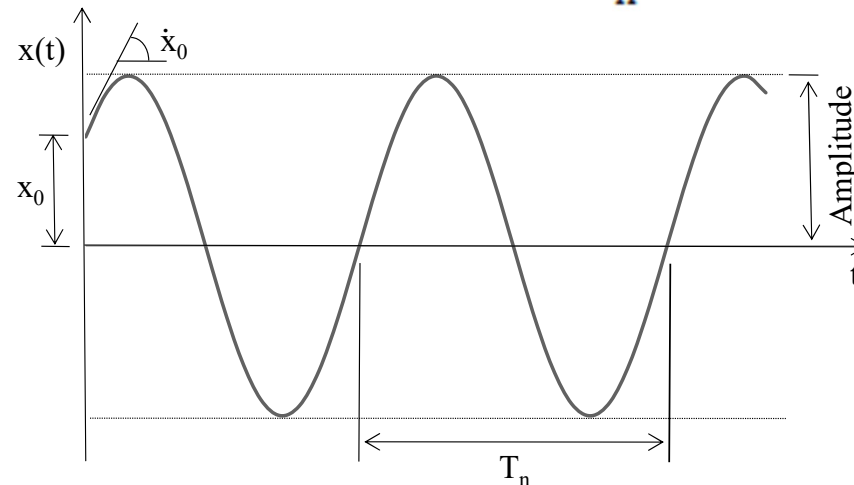
$$T_n = \frac{2\pi}{\omega_n}$$

Período
Natural

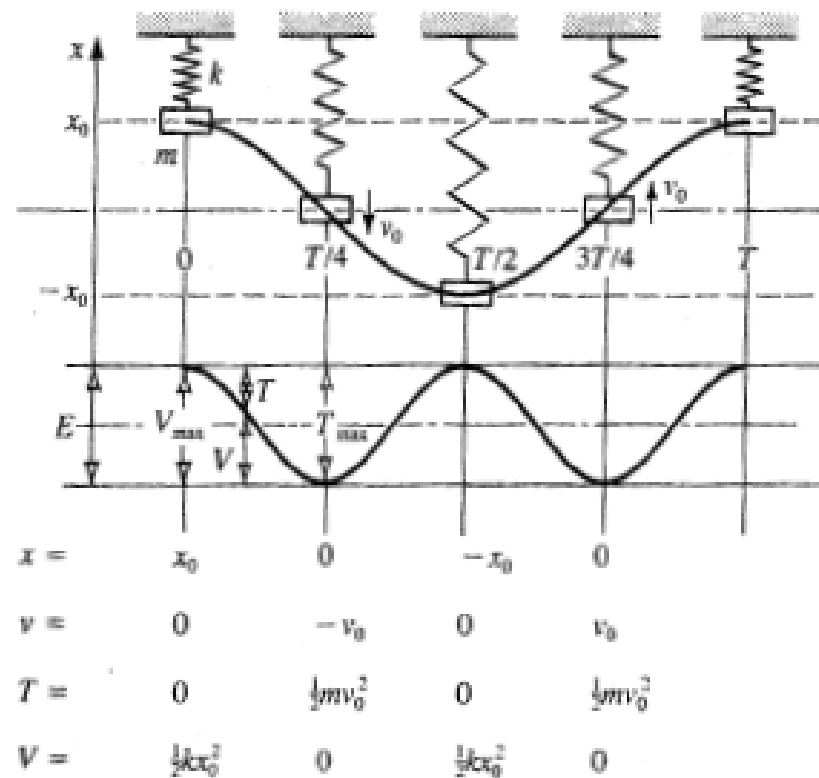
$$f_n = \frac{1}{T_n} = \frac{\omega_n}{2\pi}$$

Frequência
Natural

$$x(t) = x_0 \cos(\omega_n t) + \frac{\dot{x}_0}{\omega_n} \sin(\omega_n t)$$



R #rvflogruqlr dp ruwhflgr p rrwudgr qd iljxud whp #xp d dp solwgh#₃ 1#
D #srvtflr p rrwud xp #flr lqwhlur frp #xdwv whp srvtgh#W 27/#
d htxdflr gh#p rylp hqwr #gdgd#ru [#₃ frv+z #v,1#



Exemplo 1.1: Calcular a frequência natural da viga simplesmente apoiada de vão $l = 6,0\text{m}$, mostrada na Figura E1.1, supondo em sua seção central um equipamento com massa de $m = 760\text{ kg}$ e que seu peso próprio seja desprezível para este problema. A viga é composta por um perfil de aço, de módulo de elasticidade $E=205\text{GPa}$, com momento de inércia $I=12258\text{cm}^4$.

Solução:
$$\frac{(7.44\text{kN})(600)^2}{(48 \cdot 20500\text{kN/cm}^2 \cdot 12258\text{cm}^4)} = 0,13\text{cm}$$

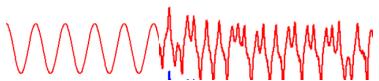
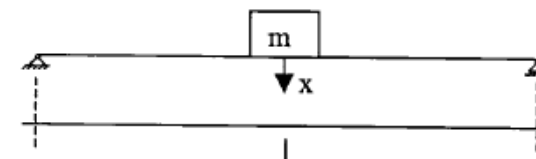
O deslocamento da seção média da viga devido a ação de uma força P nela aplicada é dada por: $\delta = \frac{Pl^3}{48EI}$.

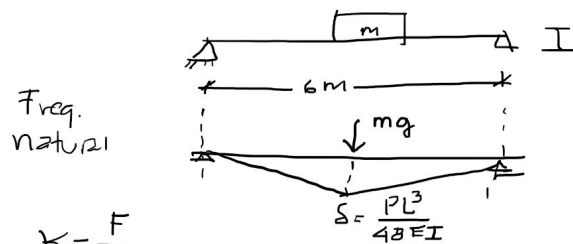
A constante de mola equivalente é dada por:

$$k = \frac{P}{\delta} = \frac{48EI}{l^3} = 5,584 \cdot 10^6 \text{ (N/m)}$$

Obtendo-se para as frequências circular e natural respectivamente:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5,584 \cdot 10^6}{760}} = 85,7\text{rad/s} ; f_n = \frac{\omega_n}{2\pi} = 13,64\text{Hz}$$





$$I = 12258 \text{ cm}^4 \rightarrow 12258 \frac{\text{cm}^4}{100^4} \frac{\text{m}^4}{\text{m}}$$

$$m = 760 \text{ kg}$$

$$E = 205000 \text{ MPa} = 2.05 \times 10^{11} \text{ N/m}^2$$

$$K = \frac{F}{\delta}$$

$$F = mg$$

$$\delta = \delta$$

$$K = \frac{mg}{\delta} = \frac{48EI}{L^3} \rightarrow K = \frac{48EI}{L^3}$$

$$K = \frac{48(2.05 \times 10^{11} \text{ N/m}^2) \frac{12258}{100^4} \text{ m}^4}{(\text{m})^3}$$

$$K = 5584200 \text{ N/m}$$

Smith

$$\omega_n = \sqrt{\frac{K}{m}}$$

$\swarrow \text{kg/m}$

$\omega_{310 \times 31}$

h

$$\omega_n = \sqrt{\frac{5584200 \text{ N/m}}{760 \text{ kg}}} = 85.7 \frac{\text{Rad}}{\text{seg}}$$

$$\frac{\text{N/m}}{\text{kg}} = \frac{\text{kg} \cdot \text{m/s}^2 / \text{m}}{\text{kg}} = \frac{1}{\text{s}^2}$$

$f_n = 12.27 \text{ Hz}$

considerando massa viz

$$f_n = \frac{1}{2\pi} \omega_n = \frac{85.7 \frac{\text{Rad/s}}{2\pi \text{ Rad}}}{2\pi}$$

$$f_n = 13.64 \text{ Hz} = 13.64 \text{ Hz}$$

