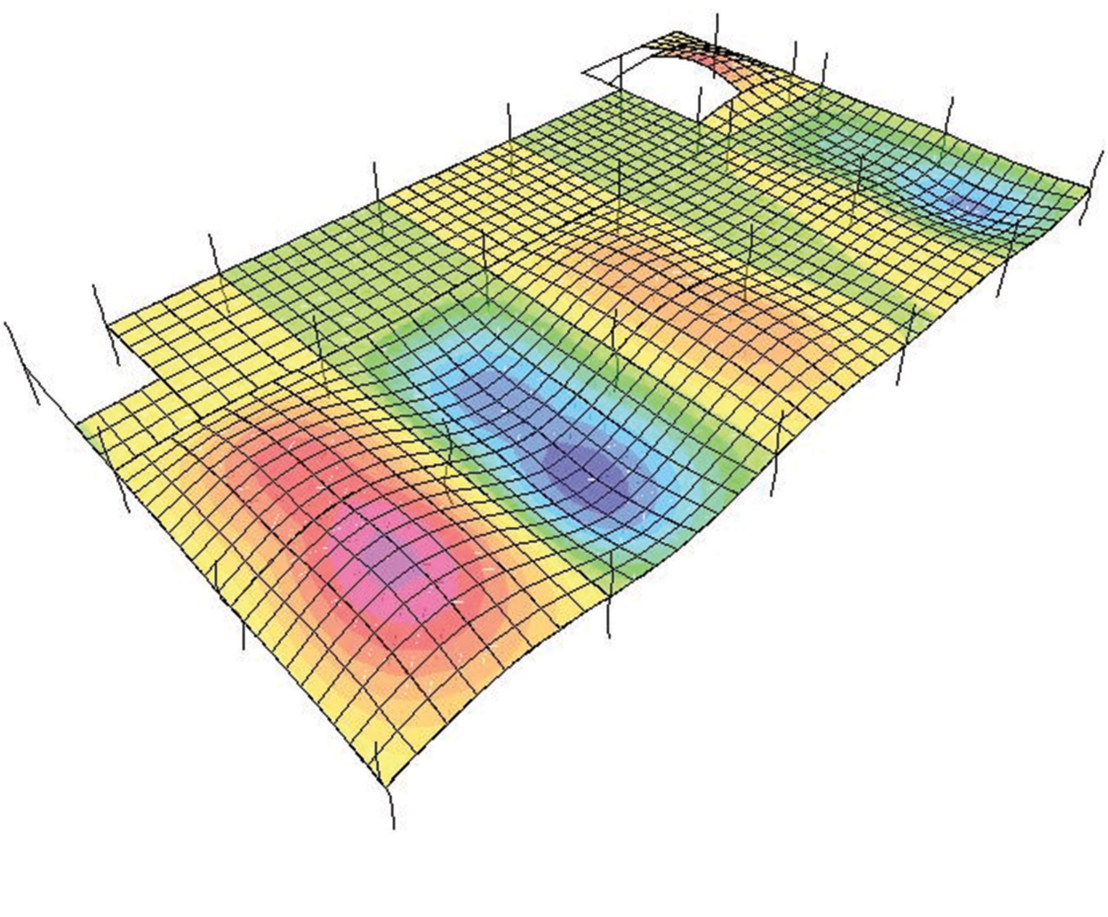
Steel Design Guide

11

Vibrations of Steel-Framed Structural Systems Due to Human Activity

Second Edition



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Vibrations of Steel-Framed Structural Systems Due to Human Activity

Second Edition

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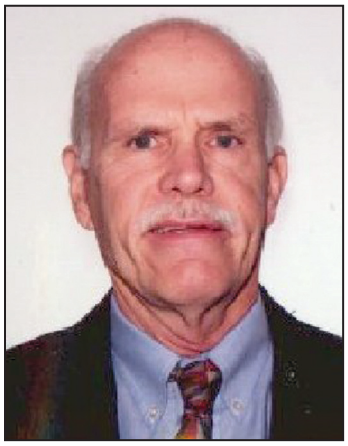
Preface

This is the second edition of Design Guide 11. The first edition was published in 1997 as *Floor Vibrations Due to Human Activity*. The scope of this edition has been broadened as reflected in the new title, *Vibrations of Steel-Framed Structural Systems Due to Human Activity*. Since 1997, a large volume of literature has been published on the response of steel-framed structural systems, including floors, monumental stairs and balconies due to human activity. Some human tolerance and sensitive equipment tolerance limits have been modified, and updated methods to evaluate high-frequency systems have been proposed. The use of the finite element method to analyze structural systems supporting human activity has been refined. Also, simplified methods to evaluate problem floors have been proposed. This second edition of the Design Guide updates design practice in these areas.

DEDICATION



This edition of Design Guide 11 is dedicated to Dr. David E. Allen, retired Senior Research Officer, Institute for Research in Construction, National Research Council Canada. Dr. Allen made outstanding contributions to the first edition of this Guide but was unable to participate in the writing of this second edition because of health reasons. Dr. Allen is known worldwide for his research and writing on floor vibration serviceability. His interest in vibration of structures due to human activity began in the early 1970s, and since then he has written numerous papers on the subject. He has been the major contributor to the vibration provisions in the National Building Code of Canada and is the first author of the Applied Technology Council Design Guide 1, *Minimizing Floor Vibration*. In 2004, Dr. Allen received the Julian C. Smith Medal for Achievement in the Development of Canada. His contributions to the development of guidelines for the evaluation of floor vibration serviceability will be long remembered by the structural engineering profession.

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Chapter 1
Introduction

1.1 OBJECTIVES OF THE DESIGN GUIDE

The primary objective of this second edition remains the same as that of the first edition, i.e., to provide basic principles and simple analytical tools to evaluate vibration serviceability of steel-framed structural systems subjected to human activity. Both human comfort and the need to control the vibration environment for sensitive equipment are considered. Other objectives are to provide guidance on the use of the finite element method and on developing remedial measures for problem floors.

1.2 ROAD MAP

This Design Guide is organized for the reader to move from basic principles of occupant-caused vibration and the associated terminology in Chapter 1; to serviceability criteria for evaluation and design in Chapter 2; to estimation of natural frequency (the most important vibration property) in Chapter 3; to applications of the criteria in Chapters 4, 5, 6 and 7; and finally to possible remedial measures in Chapter 8. Chapter 4 covers walking-induced vibration of building floors, pedestrian bridges and monumental stairs. Chapter 5 concerns vibrations due to rhythmic activities such as dancing, aerobics and spectator crowd movements. Chapter 6 provides guidance on the design of floors supporting sensitive equipment, a topic requiring increased specialization. Chapter 7 provides guidance on the use of the finite element method to predict vibration response of structural systems that cannot be evaluated by the methods in Chapters 4, 5 and 6. Chapter 8 provides guidance on the evaluation of problem floors and the choice of remedial measures. Symbols definitions and References are found at the end of the Design Guide.

1.3 BACKGROUND

Vibration serviceability is a dominant consideration in the design of steel floor framing, monumental stairs, pedestrian bridges and stadia. Modern design specifications, coupled with today's stronger steels and concretes, allow for lighter sections when strength considerations govern. Monumental stairs and pedestrian bridges are longer and more slender than in past years. Balconies and grandstands in stadia have longer cantilevers and lighter seating. Office building build-out has also dramatically changed with the universal use of computers. Virtually paperless offices are lighter and have lower damping than those with large file cabinets, heavy desks, and bookcases. Modern open floor layouts with few partitions, widely spaced demountable partitions or none at all, and light furniture add to the problem. Renovated office spaces with the removal of fixed partitions have resulted in floor vibration complaints that had never been encountered previously. As a result, vibration due to human activity is a significant serviceability concern.

A traditional stiffness criterion for steel floors limits the live load deflection of beams or girders supporting "plastered ceilings" to span/360. This limitation, along with restricting the member span-to-depth ratio to 24 or less, have been widely applied to steel-framed floor systems in an attempt to control vibrations, but with limited success. A number of analytical procedures have been developed in several countries that allow a structural designer to assess a design for occupant comfort for a specific activity and for suitability for sensitive equipment. Generally, these analytical tools require the calculation of the first natural frequency and the maximum amplitude of acceleration or velocity due to a reference excitation. An estimate of damping in the floor is also required. The response is compared to the tolerance limit for human comfort or sensitive equipment, as applicable, to determine whether the system meets serviceability requirements. Some of the analytical tools incorporate limits into a single design formula whose parameters are estimated by the designer.

The analytical tools presented in this Design Guide represent years of research and have been shown to yield useful predictions of the acceptability of vibration response of steel-framed structures subject to human activity.

1.4 BASIC VIBRATION TERMINOLOGY

The purpose of this section is to introduce the reader to terminology and basic concepts used in this Design Guide.

Acceleration ratio. For the purposes of this Design Guide, the vertical acceleration at a location divided by the acceleration of gravity.

Bay. A rectangular plan portion of a floor defined by four column locations.

Beam or joist panel. A rectangular area of a floor associated with movement of its beams or joists. The area is equal to the beam or joist span times an effective width determined from the floor system structural properties.

Damping and critical damping. Damping refers to the loss of mechanical energy in a vibrating system over time. Viscous damping is associated with a retarding force that is proportional to velocity. Damping is usually expressed as the percent of critical damping, which is the ratio of actual damping (assumed to be viscous) to critical damping. Critical damping is the smallest amount of viscous damping for which a free vibrating system that is displaced from equilibrium and released comes to rest without oscillation. For damping that is smaller than critical, the system oscillates freely as shown in Figure 1-1. Damping cannot be calculated and must be determined experimentally, usually using experimental modal analysis techniques that result in detailed identification of modal properties. In some cases, it can be measured using the decay of vibration following an impact such as a heel-drop. Damping ratios for the structural systems considered in this Design Guide are usually between 1% and 8% of critical viscous damping.

Dynamic loadings. Dynamic loadings can be classified as harmonic, periodic, transient and impulsive as shown in Figure 1-2. Harmonic or sinusoidal loads are usually associated with rotating machinery. Periodic loads may be caused by rhythmic human activities such as dancing and aerobics or by machinery that generate repetitive impacts. Transient loads occur from the movement of people and include walking and running. Single jumps and heel-drop impacts are examples of impulsive loads.

Effective impulse. For the purposes of this Design Guide, an effective impulse is a mathematical representation of a human footstep. It is used to scale the unit impulse response of a single-degree-of-freedom system to the response of the system to a human footstep.

Equivalent sinusoidal peak acceleration (ESPA). Amplitude of sinusoidal acceleration that would have approximately the same effect on occupants as that of a walking

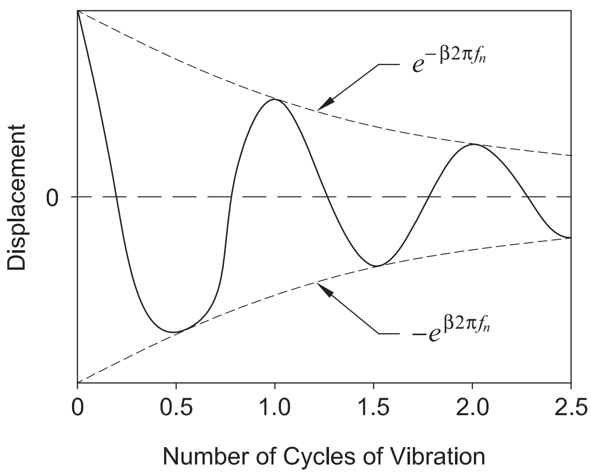


Fig. 1-1. Decaying vibration with viscous damping.

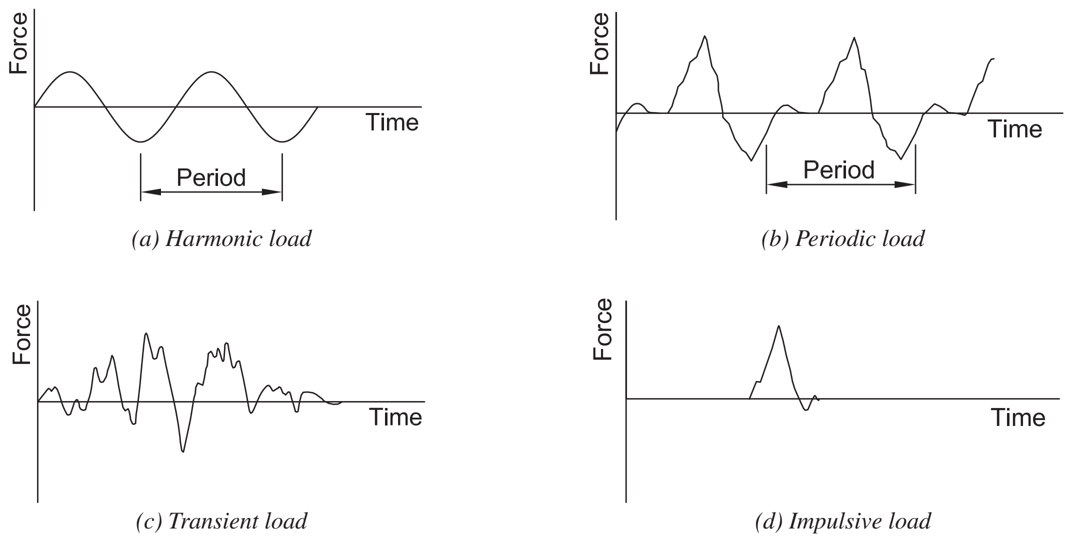


Fig. 1-2. Types of dynamic loadings.

event. Acceleration due to walking is usually transient, so is not directly comparable to typical tolerance limits that are expressed as sinusoidal accelerations. The ESPA is the product of the maximum running root-mean-square (RMS) of the walking acceleration waveform and $\sqrt{2}$, which is the ratio of peak to RMS acceleration of a sinusoid.

Experimental modal analysis (EMA). For the purposes of this Design Guide, experimental modal analysis is the process of applying a measured force to the structure, measuring the resulting accelerations, and post-processing the data to determine modal properties of the structural system.

Evaluation criterion. An inequality used to predict whether or not vibration will be objectionable. Each criterion consists of a predicted structural vibration response and a tolerance limit.

Floor length. Distance perpendicular to the span of the girders in the bay under consideration over which the structural framing (beam or joist and girder size, spacing, length, etc.) is identical, or nearly identical, in adjacent bays.

Floor panel. A rectangular plan portion of a floor encompassed by the span and an effective width or length.

Floor width. Distance perpendicular to the joist or girder span of the beams or joists in the bay under consideration over which the structural framing (beam or joist and girder size, spacing, length, etc.) is identical, or nearly identical, in adjacent bays.

Fourier series. A series of sinusoids; used in this Design Guide to represent human-induced forces. Each sinusoidal term is known as a harmonic and is characterized by its amplitude, frequency and phase lag.

Fourier transformation, fast Fourier transformation (FFT). A mathematical procedure to transform a time record into a complex frequency spectrum (Fourier spectrum) without loss of information is called a Fourier transformation. Continuous time functions and discretely sampled signals are transformed using analytical Fourier transformations and fast Fourier transformations, respectively.

Frequency response function (FRF). For the purposes of this Design Guide, a frequency response function is a plot of sinusoidal response (ratio of acceleration amplitude to force amplitude, with units of %*g*/lb) versus frequency. High FRF magnitudes indicate dominate natural frequencies.

Fundamental modal mass and effective mass. For the purposes of this Design Guide, the fundamental modal mass, also called effective mass of the structural system, is the mass of a single-degree-of-freedom system whose steady-state response to sinusoidal forcing is equal to the response of the structural system being evaluated.

Girder panel. A rectangular area of the floor associated with girder movement. The area is equal to the girder span times an effective length determined from the floor system structural properties.

Harmonic and sub-harmonic frequency. For the purposes of this document, a harmonic frequency is an integer multiple of the step frequency. For example, the harmonic frequencies of a 2-Hz step frequency are 2 Hz, 4 Hz, 6 Hz, etc. A sub-harmonic frequency is an integer subdivision of a frequency. For example, the fourth sub-harmonic frequency of an 8-Hz natural frequency is 2 Hz.

Heel-drop. A means of producing an impact on a floor, produced by a person standing on his or her toes and letting the heels drop without the toes lifting from the floor.

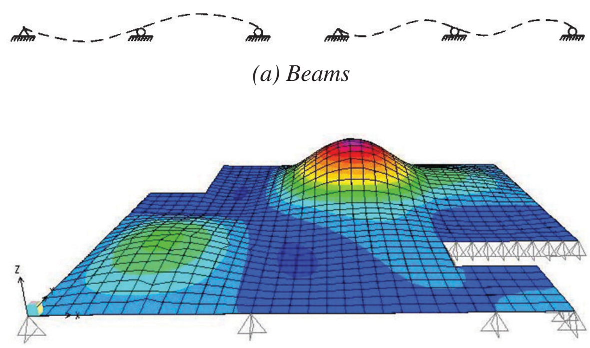
Impulse, unit impulse and unit impulse response. An impulse is a high force that acts for an extremely short time duration. Impulses are expressed using force-time units such as lb-s. When a single-degree-of-freedom system is subjected to a unit impulse, the resulting unit impulse response is characterized by an initial peak velocity proportional to the reciprocal of the mass followed by sinusoidal decay at the natural frequency.

Low- and high-frequency systems. A low-frequency system is one that can undergo resonant build-up due to the applicable human-induced dynamic loading. A resonant build-up can occur if at least one responsive natural mode has a frequency less than the maximum considered harmonic frequency. A high-frequency system is one that cannot undergo resonant build-up due to the applicable dynamic loading because all responsive frequencies are greater than the maximum considered harmonic frequency. The response of a high-frequency system resembles a series of individual impulse responses to individual footsteps.

Modal analysis. An analytical or experimental method for determining the natural frequencies and mode shapes of a structure, as well as the structural responses of individual modes to a given excitation.

Modal frequency. See natural frequency.

Mode shape, mass-normalized mode shape, and unity-normalized mode shape. When a structure vibrates freely in a particular mode, it moves with a certain deflection configuration referred to as a mode shape. Each natural frequency has a mode shape associated with it. Figure 1-3 shows typical mode shapes for a simple beam and for a slab/beam/girder floor system. A mode shape normalized such that all



(a) Beams

(b) Floor

Fig. 1-3. Typical beam and floor system mode shapes.

mass matrix entries are 1.0 is referred to as a mass-normalized mode shape. A mode shape normalized such that the maximum amplitude, usually at midbay or midspan, is 1.0 is referred to as a unity-normalized mode shape.

Narrowband spectrum. A narrowband spectrum shows vibration magnitudes in closely spaced frequency bands—usually 0.05 Hz wide. The spectral magnitude in each frequency band corresponds to the energy at frequencies within that band. Figure 1-4(a) shows an acceleration waveform. The corresponding narrowband spectrum is shown in Figure 1-4(b).

Natural frequency, free vibration, modal frequencies, fundamental natural frequency, and dominant frequency. Natural frequency is a frequency at which a body or structure will vibrate when displaced and then cleanly released. This state of vibration is referred to as free vibration. All structures have a large number of natural frequencies that are also referred to as modal frequencies; the lowest frequency is referred to as the fundamental natural frequency and generally is of most concern. The dominant frequency is the frequency with the most energy or highest response compared to all other frequencies.

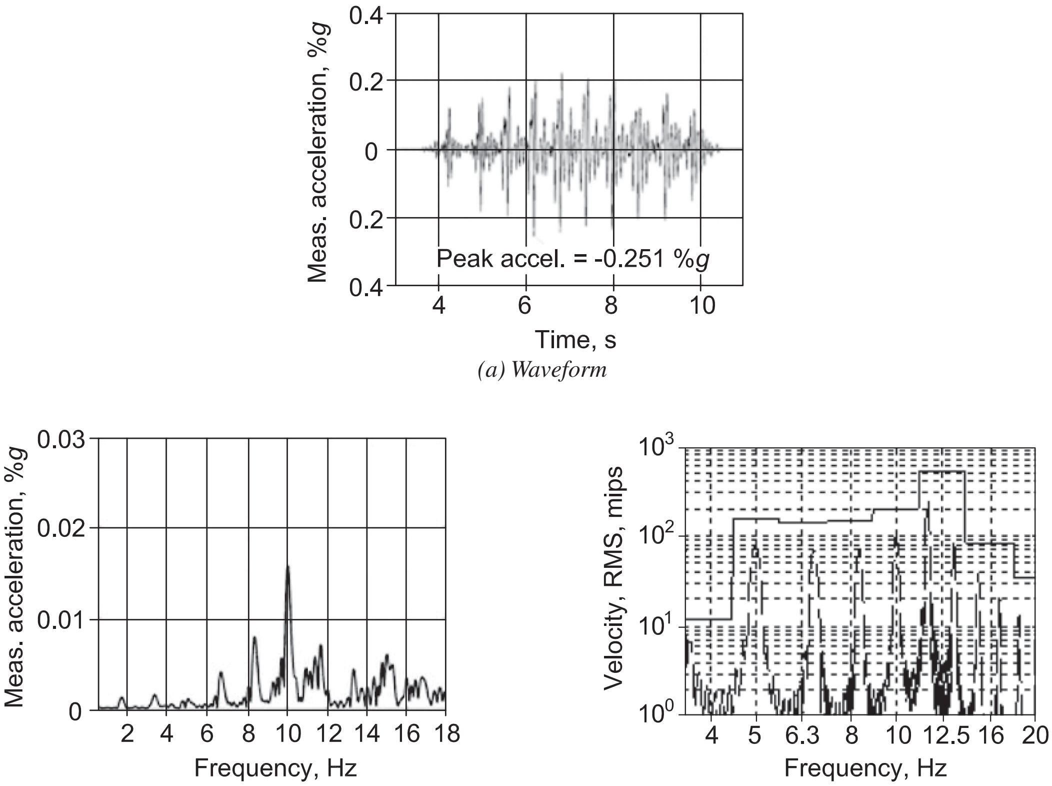
One-third octave spectrum. A one-third octave spectrum has fairly wide frequency bands identified by the following standard center frequencies in the range of typical structure vibrations: 4, 5, 6.3, 8, 10, 12.5, 16 and 20 Hz. Each band is defined by its center, lower-bound and upper-bound frequencies. For example, the lower and upper bounds of the 12.5-Hz one-third octave band are 11.2 and 14.1 Hz, respectively. Figure 1-4(c) shows an acceleration waveform and corresponding one-third octave velocity spectrum.

Period and frequency. Period is the time, usually in seconds, between successive peak excursions in uniformly repeating or steady-state events. Period is associated with harmonic (or sinusoidal) and periodic (repetitive) time functions as shown in Figures 1-2(a) and (b). Frequency is the reciprocal of period and is usually expressed in Hertz (cycles per second).

Phase lag. Phase lag describes three distinct parameters in vibration analysis: (1) The time shift of a Fourier series term, (2) The time shift of terms in a spectrum, and (3) The time shift of the sinusoidal response relative to the sinusoidal force in a frequency response function.

Resonance. If a harmonic frequency of an exciting force is equal to a natural frequency of the structure, resonance will occur. At resonance, the amplitude of the motion tends to become large to very large, as shown in Figure 1-5.

Resonant build-up. If a harmonic frequency of an exciting



(a) Waveform

(b) Narrowband acceleration spectrum

(c) One-third octave velocity spectrum

Fig. 1-4. Example waveform, narrowband spectrum, and one-third octave spectrum.

force is equal to a natural frequency of the structure that initially is at rest, the vibration will increase as shown in Figure 1-6. If the force is applied for a short duration, as with most human-induced loads, a partial resonant build-up occurs. If the force is applied for a long duration, steady-state motion is achieved.

Root-mean-square (RMS). The root-mean-square of a set of values is the square-root of the sum of the squares of these values. For a sinusoid, the root-mean-square is the peak value divided by $\sqrt{2}$.

Spectrum, spectral accelerations and spectral velocities. A spectrum shows the variation of relative amplitude, by frequency, of the vibration components of a time-history waveform, such as load or motion. Any time history waveform—such as force or acceleration—can be equivalently represented by an infinite series of sinusoids with different frequencies, magnitudes and phases. For the purposes of this Design Guide, a spectrum is a plot of these magnitudes versus frequency. Magnitudes in a spectrum have the same units as the waveform, such as lb or *%g,* and usually represent amplitudes (peak) or root-mean-square (RMS) values. Acceleration and velocity spectrum magnitudes are referred to as spectral accelerations and spectral velocities. Figure 1-7 is an example of a waveform and corresponding frequency spectrum.

Steady-state motion, transient motion and impulse response. If a structural system is subjected to a continuous harmonic load, vibration response will be sinusoidal, building up to steady-state motion as shown in Figure 1-6. If a structural system is subjected to a transient load such as a series of footsteps, vibration response will be a combination of frequency components as shown in Figure 1-8. Such vibration is referred to as transient motion. If a structural system is subjected to an impulsive load, the impulse response will consist of an initial peak response followed by decaying free vibration as shown in Figure 1-1. Individual footstep responses on a high-frequency floor, such as those in Figure 1-4(a), are approximated by impulse responses.

Step frequency. Frequency at which a foot or feet impact the supporting structure, e.g., in walking, running, dancing or aerobics.

Tolerance limit. Vibration level above which vibrations are predicted to be objectionable.

Tuned mass damper (TMD). Mass attached to a floor structure through a spring and damping device. A TMD can limit build-up of resonant vibration of a floor by transfer of kinetic energy from the floor into the TMD and dissipating some of the energy via the damping device.

Vandal or rogue jumping. For the purposes of this Design Guide, when an individual or group deliberately excites a structural system by jumping or moving the body at a subharmonic of a natural frequency of the system causing large deflection amplitudes.

Walking event. For purposes of this Design Guide, a floor velocity or acceleration waveform consisting of response to footsteps followed by freely decaying vibration until the arrival of the next walker.

Waveform. A plot of a function, such as a dynamic loading

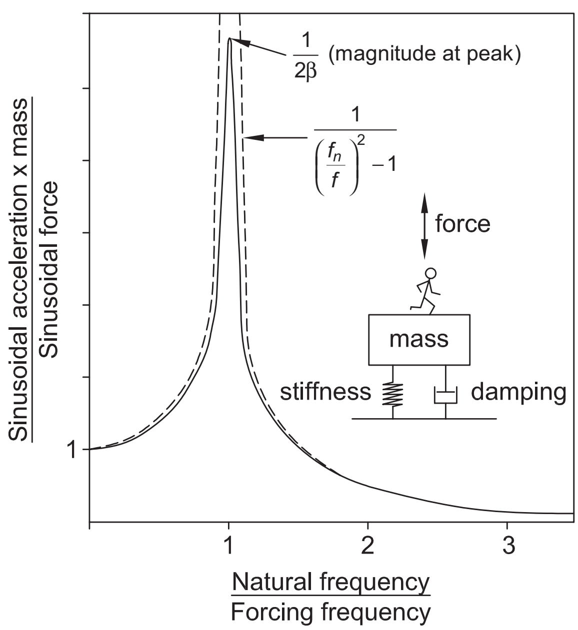


Fig. 1-5. Steady-state response of mass-spring-damper system to sinusoidal force.

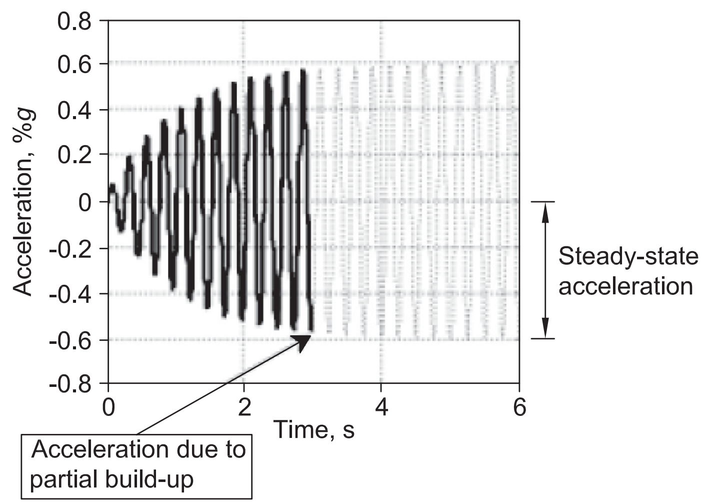


Fig. 1-6. Partial resonant build-up due to walking.

or an acceleration versus time—see Figures 1-4(a), 1-7(a) and 1-8(a). It is also known as a time history or time domain representation.

1.5 STRUCTURAL RESPONSE PRINCIPLES RELATED TO HUMAN ACTIVITY

Vibration serviceability evaluation criteria are inevitably more complicated and less precise than strength and stiffness evaluation criteria because human-induced dynamic loads vary widely and the dynamic properties of the structure are more difficult to predict than its static properties. Experience and experimental studies have shown, however, that the problem can be simplified sufficiently because the response is typically dominated by a single mode of vibration to provide practical evaluation criteria for structural systems subject to human activity. Each evaluation criterion in this Design Guide consists of two parts: a prediction of the structural response in terms of acceleration or velocity and a tolerance limit. If the structural response does not exceed the tolerance limit, then the floor or other evaluated element is predicted to allow only acceptable levels of vibration due to human activity. Basics of structural response prediction are the subject of this section. Tolerance limits for human comfort are discussed in Chapter 2 and for sensitive equipment in Chapter 6.

Human walking, dancing and other movements cause dynamic loads to be applied to the structures. These loads are periodic, or nearly so, and are fairly complicated. A number of researchers (Rainer et al., 1988; Kerr and Bishop, 2001; Brownjohn et al., 2004) show force waveforms and spectra for several types of human-induced loads. The spectra indicate significant contributions at the step frequency and its first few harmonics. Force magnitudes decrease with increasing harmonic number and are insignificant beyond the third or fourth harmonic. Thus, human-induced forces are conveniently represented by the Fourier series:

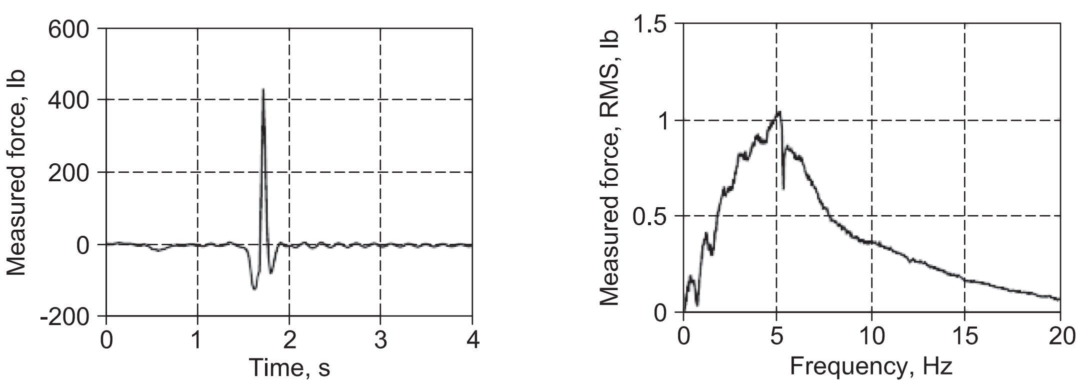
$$F(t) = Q + \sum_{i=1}^{N} \alpha_i Q \sin(2\pi i f_{step} t - \phi_i) \qquad (1\text{-}1)$$

where

N = number of considered harmonics, i.e., the number of harmonics with significant amplitudes
Q = bodyweight, lb
f_{step} = step frequency, Hz
i = harmonic number
t = time, s
α_i = dynamic coefficient (ratio of harmonic force magnitude to bodyweight) for the ith harmonic
ϕ_i = phase lag for the ith harmonic, rad

A response prediction method begins with a prediction of the natural vibration modes, each of which is characterized by its natural frequency, mode shape, damping and modal mass. Classical vibration theory or finite element analysis (FEA) is used to predict the natural frequencies and mode shapes. An example floor mode shape is shown in Figure 1-3(b). Damping—which depends mostly on the presence of nonstructural elements such as ceilings, mechanical equipment, partitions and furnishings—is estimated using guidelines based on experience. When the structure is forced at a natural frequency, its responsiveness is inversely proportional to the modal damping. It is also inversely proportional to the corresponding modal mass, which depends on the uniform mass and the extent to which the motion shares in the total mass. Finally, the response is directly proportional to the mode shape values at the dynamic force and response locations.

The fundamental natural frequency determines the type of vibration response to human activities. If the fundamental frequency is low enough to be matched by a force harmonic



(a) Heel-drop dynamic force waveform

(b) Spectrum

Fig. 1-7. Example waveform and corresponding spectrum.

frequency, then the maximum response will be a resonant build-up. A system that allows such a response is referred to as a low-frequency system, typically less than 9 Hz. As shown in Figures 1-5 and 1-8, resonant responses can result in very high accelerations. Such responses cause nearly all vibration serviceability problems related to human comfort. If the fundamental frequency is greater than the maximum considered harmonic frequency, resonance is not possible, and the response will resemble a series of impulse responses, each with an initial peak followed by decay until the next footstep is applied. Figure 1-4(a) shows an example series of individual footstep impulse responses. A system with a fundamental frequency high enough to preclude resonant responses and thus only allow individual impulse responses is referred to as a high-frequency system. High-frequency responses are almost always relatively small but can still cause problems when the tolerance limit is stringent, as is often the case with sensitive equipment.

Figure 1-8 illustrates a typical resonant build-up due to walking on a structure with a 5-Hz fundamental frequency, where the step frequency, 1.67 Hz, is such that the third harmonic frequency (three times the step frequency) matches the fundamental frequency. The spectrum indicates small responses to the first, second and fourth harmonics. The vast majority of the response is due to resonance caused by the third harmonic. It is typical for one force harmonic to match a natural frequency—very often the fundamental frequency—and to contribute almost all of the response. Thus, the resonant response can be approximated by the response of the fundamental mode, which is easy to compute using classical vibration theory. Equation 1-2 gives the steady-state acceleration, $a_{steadystate,}$, of a single-degree-of-freedom system with mass, M, and damping, β, subjected to a sinusoidal load with amplitude, P, and frequency matching the natural frequency, f_n. This equation, with $P = \alpha_h Q$, where α_h is the dynamic coefficient for the force harmonic causing resonance and fundamental modal mass is used for M, is the basis for most resonant response predictions in modern occupant-induced vibration evaluation criteria. Occasionally, such as with rhythmic group activities, the nonresonant response to a sinusoidal load at frequency, f_{step}, is required, and Equation 1-3 gives the corresponding steady-state acceleration. Note that Equation 1-3 reduces to Equation 1-2 when $f_n = f_{step}$.

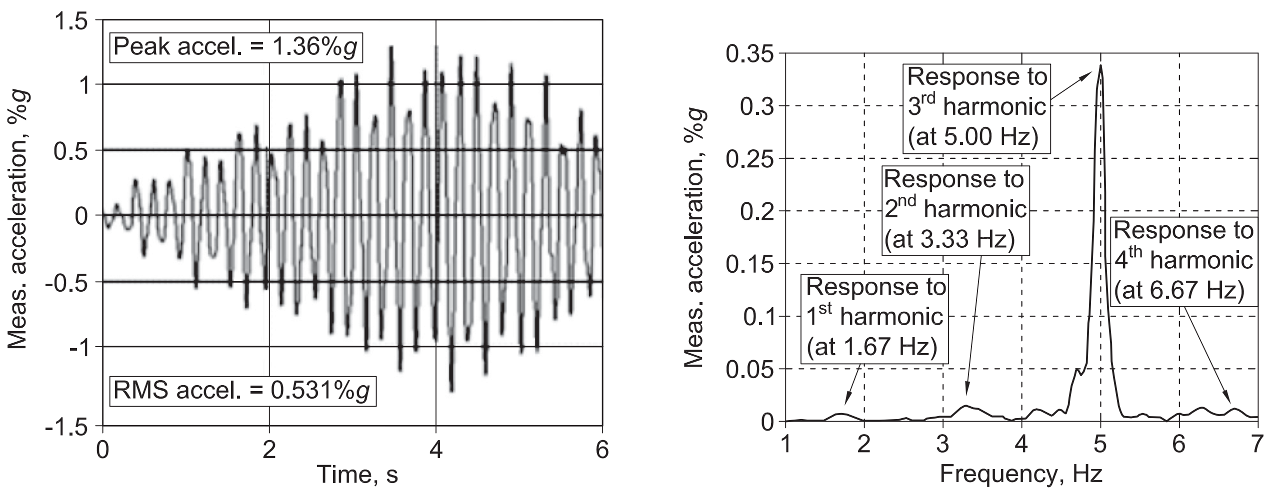
$$a_{steadystate} = \frac{P}{2\beta M} \tag{1-2}$$

$$a_{steadystate}(f_{step}) = \frac{P/M}{\sqrt{\left[\left(\frac{f_n}{f_{step}}\right)^2 - 1\right]^2 + \left(2\beta\frac{f_n}{f_{step}}\right)^2}} \tag{1-3}$$

where

M = fundamental modal mass, lb-s^2/in.
P = amplitude of sinusoidal load, lb
$a_{steadystate}$ = steady-state acceleration, in./s^2
f_n = fundamental natural frequency, Hz
f_{step} = step frequency, Hz
β = modal damping ratio

Figure 1-4(a) shows a typical series of individual responses to footsteps on a high-frequency floor. Each individual footstep response resembles an impulse response with an initial peak followed by decay. Kerr (1998) and Willford et al. (2006) have used experimental footstep measurements to develop an effective impulse, I_{eff}, which results in the same



(a) Walking dynamic force waveform due to walking at 1.67 Hz

(b) Spectrum

Fig. 1-8. Resonant response due to walking.

Table 1-1. Fourier Series Parameters for Individuals					
Activity	**Source**	***Q*, lb**	**f_{step} Range, Hz**	**Dynamic Coefficients, α_i**	**Phase Lag, ϕ_i, radians**
Walking	Rainer et al. (1988) Allen and Murray (1993)	157	1.6–2.2	0.5, 0.2, 0.1, 0.05	—
	Willford et al. (2007) Smith et al. (2007) Davis and Murray (2010)	168	1.6–2.2	0.4, 0.07, 0.06, 0.05	0, −π/2, π, π/2
Running	Rainer et al. (1988)	*	1.6–4.0	1.4, 0.4, 0.2, 0.1	—
	Bachmann et al. (1995)	*	2.0–3.0	1.6, 0.7, 0.2	—
	ISO (2007)	*	2.0–4.0	1.4, 0.4, 0.1	—
Stair descent	Kerr and Bishop (2001) Davis and Murray (2009) Davis and Avci (2015)	168	1.6–4.0	1.1, 0.2, 0.09, 0.06	—

*Depends on running event: 150 to 175 lb for recreational runners; 250+ lb for American football or rugby players.

average velocity as that caused by an actual footstep. It is proportional to bodyweight and step frequency and inversely proportional to natural frequency, and has lb-s units. Because the velocity of a single-degree-of-freedom system immediately after the application of a unit impulse is the reciprocal of the mass, *M*, of the system, the peak velocity after a footstep application obeys Equation 1-4. The corresponding peak of the system acceleration just after the footstep application is approximated using Equation 1-5. This equation with the fundamental modal mass used for *M* is the basis for impulse response predictions in vibration evaluation criteria for high-frequency systems.

$$v(\tau) = \frac{I_{eff}}{M} \tag{1-4}$$

$$a_p = \left(2\pi f_n\right)\left(\frac{I_{eff}}{M}\right) \tag{1-5}$$

where
I_{eff} = effective impulse, lb-s
Q = bodyweight, lb
a_p = peak acceleration, in./s^2
τ = time immediately after a footstep application, s

and

$$I_{eff} = \left(\frac{f_{step}{}^{1.43}}{f_n{}^{1.30}}\right)\left(\frac{Q}{17.8}\right) \tag{1-6}$$

1.6 WALKING, RUNNING AND RHYTHMIC FORCING FUNCTIONS

Human-induced dynamic loads for walking and running have been measured and reported by several researchers. In most of these experimental programs, subjects walked across instrumented structures or platforms that measured force waveforms. The measured waveforms themselves are not useful for the development of response prediction equations, such as in Chapters 4, 5, 6 and 7, because they are complicated functions of time for which no closed-form solution exists. Also, force waveforms cannot be used in response history analyses to predict the structural response because small changes in waveform often result in large changes in force spectrum and thus large changes in predicted vibration response. For these reasons, waveforms must be converted to mathematical representations useful toward predicting the vibration response. A Fourier series is a summation of sinusoids and is the preferred force representation for resonant response predictions. An effective impulse is the preferred representation for single footstep peak response predictions.

Fourier series for various individual activities have been developed as follows. Measured waveforms were Fourier transformed to corresponding spectra such as the one shown in Figure 1-8(b). Values at harmonic frequencies provided estimates of the dynamic coefficient for each significant force harmonic. Experiments also provided estimates of the expected range of step frequencies. In a few cases, phase lags were determined. The Fourier series in Equation 1-7 is formed from the experimentally obtained dynamic coefficients, step frequency range, and phase lags. The step frequency, f_{step}, that causes the maximum response is selected within the range so that one harmonic frequency matches a natural frequency and causes resonance. (Phase lags are only used in Chapter 7.) Equation 1-7 is similar to Equation 1-1, but without the constant term, which represents static force and is not used in the prediction of vibrational structural response. Table 1-1 summarizes Fourier series parameters for several common dynamic loads applied by individual walkers and runners. Note that the walking load parameters

Table 1-2. Fourier Series Parameters for Groups					
Activity	**Source**	w_p[a], **psf**	f_{step} **Range, Hz**	**Dynamic Coefficients,** α_i	**Phase Lag,** ϕ_i, **rad**
Group dancing	NRCC (2010a)	12.5	1.5–2.5	0.5, 0.05	—
Lively concert or sports event	NRCC (2010a)	31.0	1.5–3.0	0.25, 0.05	—
Aerobics	NRCC (2010a)	4.2	2.0–2.75	1.5, 0.6, 0.1	
Normal jumping	Smith et al. (2007)	3.9[b]	1.5-2.8	1.8, 1.3, 0.7, 0.2	$\pi/6, -\pi/6, -\pi/2, -5\pi/6$

a See Table 5-2 for assumed area per person or couple.

b Because of lack of coordination of large groups, reduction factors ($\alpha_1 = 1.61p^{-0.082}$, $\alpha_2 = 0.94p^{-0.24}$, $\alpha_3 = 0.44p^{-0.31}$), where p is the number of participants in the activity ($2 \le p \le 64$), may be applied to w_p.

by Rainer et al. (1988) are used in Chapter 4 and the ones by Willford et al. (2007) and Smith et al. (2007) are used in Chapters 6 and 7. It is noted that the Rainer et al. parameters are included in the data set used to generate the Willford et al. and Smith et al. parameters. They are listed separately in the table because they were used to develop the Chapter 4 criterion, which very accurately predicts acceptability (Pabian et al., 2013).

$$F(t) = \sum_{i=1}^{4} Q\alpha_i \sin(2\pi i f_{step} t - \phi_i) \qquad (1\text{-}7)$$

For rhythmic activities, the Fourier series is specialized for group uniform dynamic loads, $P(t)$, in psf units, as follows:

$$P(t) = \sum_{i=1}^{N} w_p \alpha_i \sin(2\pi i f_{step} t - \phi_i) \qquad (1\text{-}8)$$

where w_p is the best estimate of the unit weight of rhythmic activity participants distributed over the floor bay in psf, determined using the anticipated bodyweight and spacing of rhythmic activity participants. Recommended parameters for use in Equation 1-8 are shown in Table 1-2.

1.7 USE OF FINITE ELEMENT ANALYSIS

Vibration response prediction by finite element analysis is needed when the structural system or dynamic loads fall outside the limitations of the manual calculation methods described in Chapters 4, 5 or 6. Examples are very irregularly framed areas, areas with significant cantilevers, bays supporting localized large masses, some monumental stairs, balconies, and grandstands in stadia. Recommended finite element analysis modeling; analysis procedures; and example calculations for floors, monumental stairs and balconies are in Chapter 7. For the analysis of grandstands in stadia the reader is referred to *Dynamic Performance Requirements for Permanent Grandstands Subject to Crowd Motion* (IstructE, 2008).

Chapter 2
Evaluation Criteria for Human Comfort

This chapter describes the development of the evaluation criteria for human comfort that are implemented in Chapters 4 and 5. It also includes evaluation criteria for walking on high-frequency floors and running on a level surface.

2.1 SOURCES OF TOLERANCE LIMITS

Human response to structural motion is a very complex phenomenon involving the magnitude of the motion, the environment surrounding the sensor and the human sensor. A continuous (steady-state) motion can be more objectionable than motion caused by an infrequent impact (transient). The threshold of perception of floor motion in a busy workplace can be higher than in a quiet apartment. The reaction of a senior citizen living on the 50th floor can be considerably different from that of a young adult living on the second floor of an apartment complex, if both are subjected to the same motion.

The reaction of people who feel vibration depends very strongly on what they are doing. People in offices or residences do not like "distinctly perceptible" vibration (peak acceleration above about 0.5% of the acceleration of gravity, 0.5%g), whereas people taking part in an activity will accept vibrations 10 to 30 times greater (5% to 15%g or more). People dining beside a dance floor, lifting weights beside an aerobics gym, or standing in a shopping mall or on an indoor pedestrian footbridge will accept something in between (about 1.5%g). People on an outdoor pedestrian bridge or monumental stair will tolerate higher accelerations as well. Sensitivity within each occupancy also varies with duration of vibration and remoteness of source. It is noted that these limits are for vibration frequencies between 4 and 8 Hz, which is the range of resonance frequencies of human internal organs. Outside this frequency range, people accept higher accelerations.

The International Standards Organization ISO 2631-2, *Evaluation of Human Exposure to Whole-Body Vibration—Part 2: Human Exposure to Continuous and Shock-Induced Vibrations in Buildings (1 to 80 Hz)* (ISO, 1989) and ISO 10137, *Bases for Design of Structures—Serviceability of Buildings and Walkways Against Vibrations* (ISO, 2007) contain a baseline curve for human response to continuous sinusoidal accelerations in one-third octave bands. Recommendations for vibration tolerance limits in the form of multiplying factors applied to the base curve, e.g., 4 for office environments subject to intermittent vibration (such as from pile-driving) and 60 to 128 for transient vibration (such as from blasting) are also in ISO 2631-2. According to ISO 10137, "continuous vibrations" are those with a duration of more than 30 minutes per 24 hours; "intermittent vibrations" are those of more than 10 events per 24 hours. The standard does not contain a multiplying factor for walking vibration. Allen and Murray (1993) reasoned that a walking event causes a response that is intermittent, but not as repetitive as other intermittent responses, such as those due to pile-driving. They estimated that a reasonable range of multiplying factors for walking in an office is "5 to 8, which corresponds to a root-mean-square (RMS) acceleration in the range of 0.25 to 0.4%g." Allen and Murray used an estimated crest factor (ratio of peak to RMS acceleration) of 1.7 to convert from RMS to peak acceleration. Finally, using experience, they settled on a sinusoidal peak acceleration limit of 0.5%g for offices for vibration in the range of 3 to 10 Hz. They used similar reasoning to arrive at less stringent limits for other environments such as shopping malls. These limits were recommended in the first edition of this Design Guide and again in this edition, and are shown in Figure 2-1.

The Steel Construction Institute (SCI) P354, *Design of Floors for Vibration: A New Approach* (Smith et al., 2007), has two methods for the evaluation of floors. The continuous vibration evaluation method includes vibration limits based on SCI 076, *Design Guide on the Vibration of Floors* (Wyatt, 1989) and the British Standard BS 6472-1, *Guide to Evaluation of Human Exposure to Vibration in Buildings (1 Hz to 80 Hz)* (BSI, 1992). BS 6472-1 includes a baseline curve similar to that given in ISO 2631-2 and multiplying factors for "low probability of adverse comment." These limits are approximately half those that were being successfully used in North America during that time period; therefore, Wyatt proposed a set of multiplying factors that were half as stringent for offices or residences during daytime use. For offices, their recommended multiplying factor is 8.0, which corresponds to a peak acceleration of 0.58%g. Smith et al. (2007) adopted these multiplying factors in SCI P354 and state "To the authors' knowledge, no adverse comments have been received from occupants on floors designed to those factors."

ISO 10137 and BS 6472-1 (BSI, 2008) provide an "intermittent vibration" assessment method which uses "vibration dose values" (VDVs) as a tolerance limit to quantify the combined effects of vibration amplitude, number of events in an exposure period, and duration of each event. Although this method is more suited to in-situ measurements, designer guidance for calculating VDVs is given in SCI P354. According to Ellis (2001), "for the few occasions where a qualitative assessment has been undertaken it [VDV] appears to provide results which align with people's perception of the floors' vibrational response."

The Concrete Centre *CCIP-016: A Design Guide for Footfall Induced Vibration of Structures* (Willford and Young, 2006) quantifies tolerance limits using the multiplying factor approach similar to that used for the "continuous vibration" evaluation method in SCI P354.

The National Building Code of Canada (NRCC, 2010a) includes tolerance limits for floor vibration analysis. The User's Guide (NRCC, 2010b) refers to the first edition of this Design Guide for walking-caused vibration. Additionally, the User's Guide has recommended limits for rhythmic events that are incorporated in this edition of the Design Guide.

In the European "Human Induced Vibration of Steel Structures" (HIVOSS) method, the 90th percentile one footstep RMS (OS-RMS$_{90}$) is computed for a large set of loads "representing all possible combinations of persons' weights and walking speeds" (RFCS, 2007a; RFCS, 2007b). The OS-RMS$_{90}$ is used to determine the floor vibration classification, which indicates the types of occupancies that can be supported without objectionable vibrations.

Recommended tolerance limits for slender stairs are found in Bishop et al. (1995), Davis and Murray (2009), and Davis and Avci (2015). The recommendations are based on measurements and consider both single-person and group descending stair loadings. Recommended sinusoidal peak accelerations vary from 1.7%*g* to 4.6%*g*, depending on the loading.

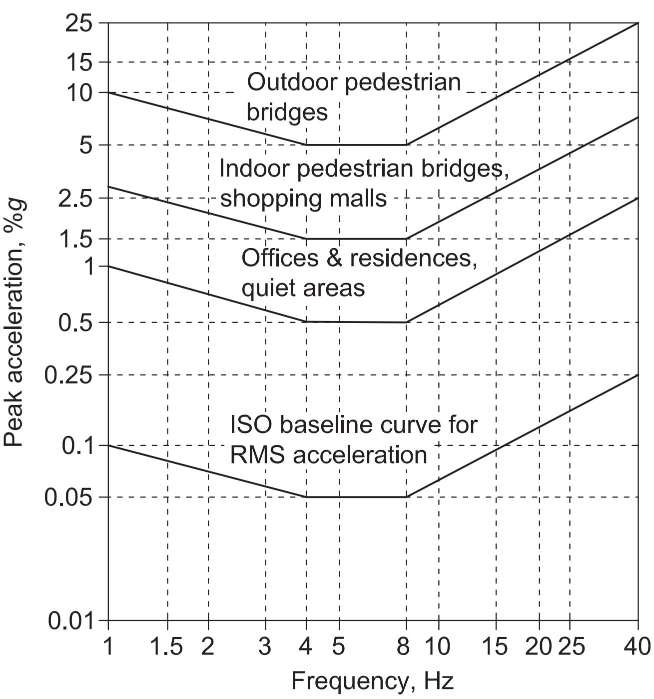


Fig. 2-1. Recommended tolerance limits for human comfort.

Recommended tolerance limits for stadia are found in *Dynamic Performance Requirements for Permanent Grandstands Subject to Crowd Motion* (IStructE, 2008) and Browning (2011).

Peak acceleration of floors, footbridges and tracks should be compared to the limits shown in Figure 2-1, as the authors are not aware of running-induced vibration tolerance limits.

Recommended tolerance limits for sensitive equipment and sensitive occupancies are found in Chapter 6 of this Design Guide.

2.2 WALKING EXCITATION—FLOORS AND PEDESTRIAN BRIDGES

2.2.1 Low-Frequency (< 9 Hz) Floors and Pedestrian Bridges

The recommended walking excitation criterion, methods for estimating the required floor properties, and design procedures for low-frequency (< 9 Hz) floors and pedestrian bridges were first proposed by Allen and Murray (1993), and included in the first edition of this Design Guide, and are recommended in this edition.

The evaluation criterion is based on the dynamic response of steel beam- or joist-supported level systems to walking forces, and can be used to evaluate structural systems supporting offices, shopping malls, schools, churches, assembly areas, pedestrian bridges and similar occupancies. Its development is explained in the following paragraphs and its application is shown in Chapter 4.

Because response to walking is often dominated by one mode, the response prediction equation is the same as that for an equivalent single-degree-of-freedom system idealized as shown in Figure 2-2. The steady-state acceleration response is given by

$$a_{steadystate} = \frac{P}{2\beta M} \tag{2-1}$$

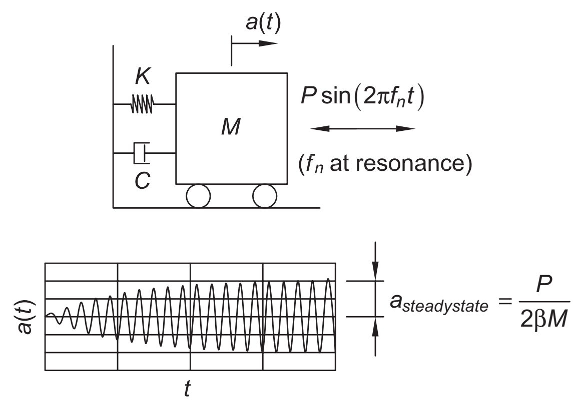


Fig. 2-2. Idealized single-degree-of-freedom system.

Table 2-1. Walking Forcing Frequencies and Dynamic Coefficients

Harmonic i	Person Walking	
	if_{step}, Hz	α_i
1	1.6–2.2	0.5
2	3.2–4.4	0.2
3	4.8–6.6	0.1
4	6.4–8.8	0.05

where

M = fundamental modal mass, lb-s^2/in.

P = amplitude of the driving force, lb

β = damping ratio

A series of footstep forces as shown in Figure 2-3 is represented by a specialized Fourier series for human-induced forces in Equation 1-1. However, only one harmonic component of Equation 1-1 is used for design because all others produce small vibrations in comparison to the harmonic associated with resonance as shown in Figure 1-8. Because only one term is used, phase lag is not considered. The first term in Equation 1-1, Q, representing the static weight of the walker, which is already on the floor when walking commences, does not need to be considered.

Therefore, Equation 1-1 reduces to

$$F(t) = \alpha_h Q \sin(2\pi h f_{step} t) \tag{2-2}$$

where

h = number of the harmonic that causes resonance

Recommended dynamic coefficients, α_i, from Rainer et al. (1988) and Allen and Murray (1993) are given in Table 2-1. A bodyweight of 157 lb was used by the researchers.

Walking step frequencies range between 1.6 and 2.2 Hz with the average range being approximately 1.9 to 2.0 Hz. As shown in Table 2-1, the maximum harmonic frequency is near 9 Hz. Thus, resonance due to walking is not possible if the floor natural frequency is above this frequency.

The amplitude, $\alpha_h Q$, from Equation 2-2 is substituted for P in Equation 2-1. Also, a reduction factor, R, is introduced to account for incomplete resonant build-up from walking (i.e., full steady-state resonant motion may not be achieved), and that the walker and the potentially annoyed person are not simultaneously at the same location of maximum modal

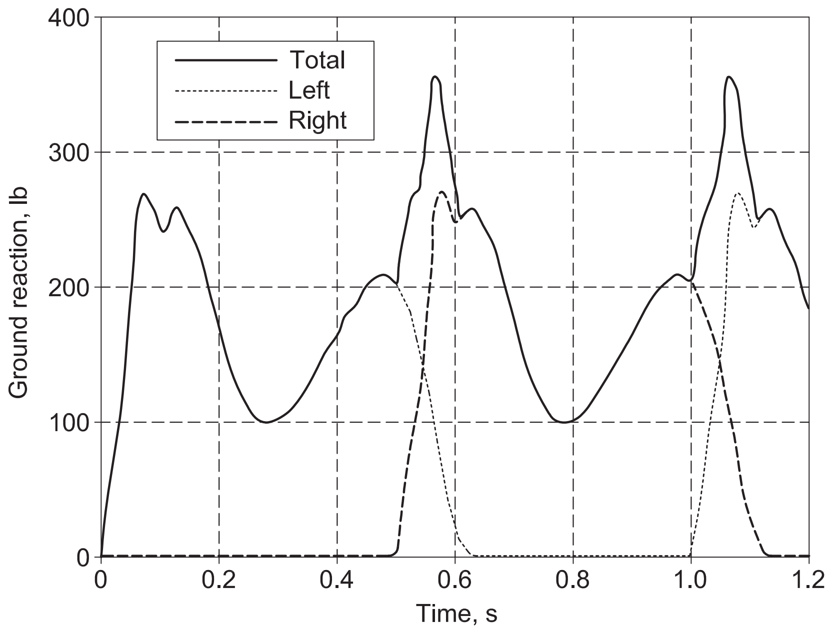


Fig. 2-3. Series of footstep forces.

displacement and acceleration. The peak acceleration is thus given by

$$a_p = \frac{R\alpha_h Q}{2\beta M} \tag{2-3}$$

where

R = reduction factor
W = effective weight of the floor, lb
a_p = peak acceleration, in./s^2

In the first edition of this Design Guide, it was recommended that R be conservatively taken as 0.5 for floor structures with two-way modal shape configurations and 0.7 for one-way modal shape configurations, such as pedestrian bridges. These values are subjective and reflect the relative differences that may occur between offices, etc., and pedestrian bridges.

For evaluation, the peak acceleration due to walking can be estimated from Equation 2-3 by selecting the lowest harmonic, h, for which $f = hf_{step}$ matches a natural frequency of the floor structure as explained in Section 1.5. The peak acceleration is then compared with the appropriate limit in Figure 2-1. For design, Equation 2-3 can be simplified by approximating the step relationship between the second and fourth harmonic dynamic coefficients, α_i, and floor natural frequency, f_n, as shown in Figure 2-4 by

$$\alpha = 0.83e^{-0.35f_n} \tag{2-4}$$

where

f_n = floor natural frequency
α = dynamic coefficient

Substituting into Equation 2-3, the predicted peak acceleration is

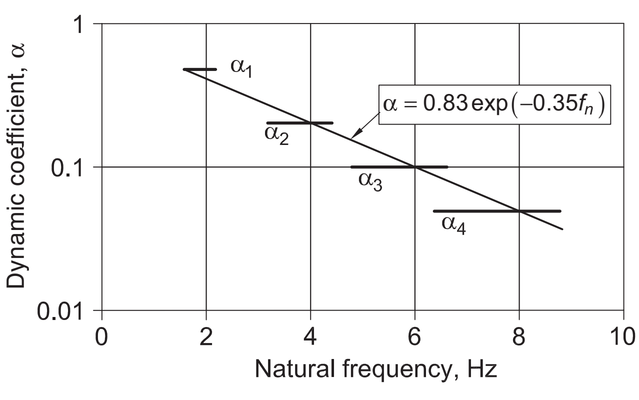


Fig. 2-4. Dynamic coefficient, α, versus natural frequency.

$$\begin{aligned} a_p &= \frac{RQ\alpha_h}{2\beta M} \\ &= \frac{RQ(0.83e^{-0.35f_n})}{2\beta\,(0.5W)/g} \\ &= \frac{R(157\text{ lb})(0.83e^{-0.35f_n})}{\beta W}g \\ &= \frac{P_o e^{-0.35f_n}}{\beta W}g \end{aligned} \tag{2-5}$$

The fundamental modal mass of a simply supported beam with uniform mass is half the beam's total mass. Thus, the 0.5 factor is required to bridge between the single-degree-of-freedom model and floor bays (Allen and Murray, 1993). Note that Equation 2-5 is not applicable to irregular framing or cantilevers.

The simplified design criterion for walking is then

$$\frac{a_p}{g} = \frac{P_o e^{-0.35f_n}}{\beta W} \le \frac{a_o}{g} \tag{2-6}$$

where

P_o = amplitude of the driving force, lb
= 0.83RQ, lb
= constant force equal to 65 lb for floors and 92 lb for pedestrian bridges
Q = bodyweight, lb
= 157 lb as recommended by Allen and Murray (1993)
a_o/g = vibration tolerance acceleration limit, expressed as an acceleration ratio (Figure 2-1)
a_p/g = ratio of the peak floor acceleration to the acceleration of gravity
f_n = fundamental natural frequency, Hz

In a recent study by the authors, Inequality 2-6 was used to predict the acceptability of each bay in a large database of steel-framed floor bays. It correctly predicted that 28 of 29 (96.6%) of the floors rated acceptable by the occupants would be acceptable and that 74 of 76 (97.4%) of the floors rated unacceptable would be unacceptable (Pabian et al., 2013).

Guidelines for the estimation of the parameters used in the preceding design criteria for walking vibration and application examples are found in Chapter 4.

2.2.2 High-Frequency (> 9 Hz) Floors and Pedestrian Bridges

High-frequency floors (no natural frequency below 9 Hz) do not undergo resonance due to walking; instead, the response to walking resembles a series of impulse responses to individual footsteps as shown in Figure 1-4(a). Liu and Davis (2015) used Equation 1-5 to determine the peak acceleration

Table 2-2. Harmonic Matching the Natural Frequency of High-Frequency Floors	
f_n, Hz	h
9–11	5
11–13.2	6
13.2–15.4	7

after a footstep with an adjustment factor, R_M, to account for contributions of modes above the fundamental mode, resulting in:

$$a_p = \frac{2\pi f_n R_M I_{eff}}{M} \tag{2-7}$$

where

R_M = higher mode factor = 2.0

I_{eff} = effective impulse, lb-s, from Equation 1-6

M = fundamental modal mass of the floor or pedestrian bridge, lb-s^2/in.
= $W/(2g)$

W = effective weight of the floor or weight of the pedestrian bridge, lb

All vibration tolerance limits in this Design Guide are expressed in terms of sinusoidal amplitudes, so the peak acceleration from Equation 2-7 must be converted to an equivalent sinusoidal peak acceleration (ESPA) as described in Davis et al. (2014). This is accomplished by (1) defining the waveform of the response to one footstep; (2) computing the RMS acceleration of that waveform; and (3) multiplying the RMS acceleration by $\sqrt{2}$, which is the ratio of peak acceleration to RMS acceleration for a sinusoid.

The individual footstep impulse response, which is an initial peak acceleration, a_p, followed by decay, is computed using

$$a(t) = a_p \, e^{-2\pi f_n \beta t} \sin(2\pi f_n t) \tag{2-8}$$

The RMS acceleration is found from

$$a_{RMS} = \sqrt{\frac{1}{T_{step}} \int_0^{T_{step}} [a(t)]^2 \, dt} \tag{2-9}$$

where

T_{step} = footstep period, s
= $1/f_{step}$

By evaluating Equation 2-9 with Equation 2-7, multiplying by $\sqrt{2}$ and simplifying, the ESPA ratio is found as shown in Equation 2-10. This equation, which uses the effective impulse from Equation 1-6, includes a calibration factor to adjust the prediction so as to match measured data. Measured and predicted ESPA ratios for 89 walking measurements in five bays in three steel-framed buildings were compared to determine the accuracy and conservatism of the predictions. The average ratio of measured-to-predicted (using no calibration factor, R) ESPA is 0.914 with 27% coefficient of variation. Introduction of a calibration factor R = 1.3 adjusts the prediction so that the probability of a measured ESPA exceeding the prediction is only 10%.

$$\begin{aligned}\frac{a_{ESPA}}{g} &= \frac{2\pi f_n R R_M I_{eff}}{W}\sqrt{\frac{1-e^{-4\pi h\beta}}{h\pi\beta}} \\ &= \left(\frac{154}{W}\right)\left(\frac{f_{step}^{1.43}}{f_n^{0.3}}\right)\sqrt{\frac{1-e^{-4\pi h\beta}}{h\pi\beta}}\end{aligned} \tag{2-10}$$

where

R = calibration factor =1.3

h = step frequency harmonic matching the natural frequency (Table 2-2)

Bodyweight, Q, was assumed to be 168 lb in determining I_{eff} for the calibration calculations.

The design criterion for walking on a high-frequency floor is then Inequality 2-11, which is recommended for floor bays with natural frequencies between 9 Hz and 15 Hz. The authors are not aware of any vibration serviceability problems, nor of any experimental data related to human comfort, for steel-framed floors with natural frequencies above 15 Hz.

$$\frac{a_{ESPA}}{g} \le \frac{a_o}{g} \tag{2-11}$$

where

a_o/g = tolerance limit acceleration ratio (Figure 2-1)

2.2.3 Lateral Vibration of Pedestrian Bridges

The recent lateral vibration problems of pedestrian bridges have resulted in a large volume of literature on the subject, but no single method of evaluation has been established. The AASHTO *LRFD Guide Specifications for the Design of Pedestrian Bridges* (AASHTO, 2009) has a minimum lateral frequency requirement of 1.3 Hz. There is no recommended lateral acceleration limit recommendation in the *Guide.*

2.3 WALKING AND RUNNING EXCITATIONS—MONUMENTAL STAIRS

Slender monumental stairs are sometimes vulnerable to annoying vibrations due to human activity. Stringer sizes are often very small for the span, resulting in low natural frequencies, and people can descend stairs at up to a 4-Hz step frequency without much effort. The combination of low natural frequency and high step frequency allows almost all slender stairs to be excitable by the second or third harmonic of the walking force. Harmonic forces due to descents are high compared to most other forces due to human activity, and stair damping and mass are often very low also. Finally, rapidly descending groups cause significant amplification compared to the response of one person—a situation that is peculiar to stairs due to the fixed stride length. Thus, resonant walking on such stairs can cause objectionable vertical accelerations. Some configurations with very low-frequency lateral vibration modes might allow disturbing lateral accelerations. The following evaluation criterion, based on the research by Davis and Avci (2015), is limited to stairs that are linear in plan (not kinked or curved horizontally), with or without intermediate landings along the span. Stairs outside the scope of the method can be evaluated using the finite element methods in Chapter 7.

Using background information provided by Bishop et al. (1995), Davis and Murray (2009), and Davis and Avci (2015), the recommended vertical vibration tolerance limits for human comfort of people standing on the stairs are 1.7%g for normal descents with step frequencies not more than 2.5 Hz; 3%g for rapid descents between 2.5 and 4.0 Hz, if perceptible vibrations are to be avoided (4.5%g, otherwise); and 4.5%g for rapidly descending groups if that load case is considered.

The stair is idealized as a beam of length, L, at a slope, θ, from the horizontal. Stairs with kinked stringers due to intermediate landings are modeled by a beam spanning along the diagonal between the supports (see Example 4.6). The fundamental mode is assumed to be a half sine wave over the length of the beam as shown in Equation 2-12.

$$\phi(x_s) = \sin\frac{\pi x_s}{L_s} \tag{2-12}$$

where

L_s = stair stringer length measured along the diagonal between supports, in.

x_s = distance measured along the diagonal between stair supports, in.

Thus, the fundamental natural frequency, f_n, is found as for a simply supported beam with uniform mass and flexural stiffness using Equation 3-1. The fundamental modal mass, M_s, of a simply supported stair with uniform mass is half the total mass of the stair:

$$M_s = \frac{W_s}{2g} \tag{2-13}$$

where

W_s = weight of the stair, lb

The maximum response to a stair descent is due to a partial resonant build-up associated with the force harmonic with frequency equaling the natural frequency. This harmonic is referred to as the hth harmonic. The dynamic coefficient is approximated using a curve-fit of the second through fourth harmonic coefficients from Table 1-1. The response is adjusted for response location, walker location, and anticipated duration of resonant buildup. A calibration factor, R, is included such that the predicted and measured responses are equal, on average. (This level of conservatism seems reasonable based on the authors' experience.) A detailed derivation is found in Davis and Avci (2015). The vertical peak acceleration ratio for an individual walker descending a stair is

$$\frac{a_p}{g} = 0.62e^{-\gamma f_n}\frac{RQ\cos^2\theta}{\beta W_s}\phi_R\phi_W\left(1-e^{-100\beta}\right) \tag{2-14}$$

where

Q = walker bodyweight = 168 lb

R = calibration factor
= 0.5 for $h = 2$
= 0.7 for $h = 3$ or $h = 4$

β = damping ratio (see Section 4.3 for recommended values)

ϕ_R = unity normalized mode shape value at response (potentially affected observer) location from Equation 2-12

ϕ_W = unity normalized mode shape value at the excitation (walker) location from Equation 2-12

θ = stair inclination from horizontal, measured with respect to support points, degrees

γ = 0.29 for normal descents
= 0.19 for rapid descents

In some cases, the acceleration due to a rapidly descending group must also be predicted. Based on Kerr (1998), Davis and Murray (2009), and Davis and Avci (2015), the predicted response of a rapidly moving group is three times the response predicted using Equation 2-14. Engineering judgment must be used to determine whether or not this load case must be considered in the evaluation of a stair. Also, this load case can only occur if the stair is wide—allowing for stationary people to be on the stair during a rapid group descent.

To avoid resonance with the first harmonic of a rapid descent, which has a maximum frequency of 4.0 to 4.5 Hz, it is also recommended that the vertical natural frequency be at least 5 Hz.

Table 2-3. Running Forcing Frequencies and Dynamic Coefficients		
Harmonic, *i*	**if_{step}, Hz**	**α_i**
1	1.6–4	1.4
2	4–8	0.4
3	8–12	0.2
4	12–16	0.1

Table 2-4. Rhythmic Forcing Frequencies and Dynamic Coefficients						
	Group Dancing		**Lively Concert**		**Aerobics**	
Harmonic, *i*	**if_{step}, Hz**	**α_i**	**if_{step}, Hz**	**α_i**	**if_{step}, Hz**	**α_i**
1	1.5–2.7	0.5	1.5–2.7	1.25	2.0–2.75	1.5
2	3.0–5.4	0.05	3.0–5.4	0.026	4.0–5.5	0.6
3	—	—	—	—	6.0–8.25	0.1

To the authors' knowledge, there are no recommended tolerance limits or dynamic coefficients for lateral vibration of stairs. Thus, it is recommended that the stair lateral vibration natural frequency be high enough to avoid resonance with the first harmonic of the lateral force due to a stair descent. With descending step frequencies as high as 4 Hz, the lateral force first harmonic can be as high as 2 Hz. Therefore, it is recommended that the lateral vibration natural frequency, computed using Equation 3-1, be greater than 2.5 Hz.

2.4 RUNNING ON A LEVEL SURFACE

There has been much less research on the response of floors to running than to walking, and there are no recommended acceleration limits in the literature for floors subjected to running. However, it seems reasonable to recommend the limits shown in Figure 2-1 using engineering judgment for selecting the appropriate category depending on the location of the running and the affected occupancy.

Three sources of dynamic coefficients for running with recommended step frequency ranges and dynamic coefficients are shown in Table 1-1. The first harmonic coefficients given in the three sources are very large, ranging from 1.4 to 1.6, for the frequency range 1.6 to 4 Hz. Floors or running tracks with natural frequencies in this range must, therefore, be very massive to prevent high accelerations. Thus, it is recommended that structures supporting running have all natural frequencies 4 Hz or greater. With this limitation, and proceeding as in Section 2.2, the response prediction method is developed as follows, noting that resonance build-up (as for low-frequency floors subject to walking) can occur with structure frequency as high as 16 Hz.

The running force is represented by the Fourier series in Equation 1-7 with recommended values for α_i from Rainer et al. (1988) in Table 1-1, repeated in Table 2-3.

Using the dynamic coefficients for harmonic numbers 2 through 4, the approximate step relationship between the dynamic coefficients and natural frequencies 4 Hz and greater is

$$\alpha = 1.13e^{-0.173 f_n} \tag{2-15}$$

Proceeding as in Section 2.2 with $R = 0.7$, the design criterion for running is

$$\frac{a_p}{g} = \frac{0.79Q(e^{-0.173 f_n})}{\beta W} \le \frac{a_o}{g} \tag{2-16}$$

where

Q = bodyweight for the running activity, 168 lb for recreational runners to 250+ lb for some athletes.

Example 4.5 illustrates the use of Inequality 2-16.

2.5 RHYTHMIC EXCITATION

The criteria in this Design Guide for the design of floor structures for rhythmic activities are based on those in the National Building Code of Canada (NRCC, 2010a; NRCC, 2010b) but slightly modified because of recent experience. The criteria can be used to evaluate structural systems supporting aerobics, dancing, audience participation and similar events over all or part of the floor, provided the loading function is known. Table 2-4 gives common forcing frequencies and dynamic coefficients for rhythmic activities.

The peak acceleration due to a harmonic rhythmic force, if_{step}, is obtained from Equation 1-3, where floor motion is assumed to be dominated by one mode of vibration with frequency f_n (Allen, 1990b). Equation 1-3 is modified using the 1.3 factor because the mass and load are uniform over the bay, which deforms in a half sine wave pattern (Allen et al., 1987).

$$\frac{a_{p,i}}{g} = \frac{1.3\alpha_i w_p / w_t}{\sqrt{\left[\left(\frac{f_n}{if_{step}}\right)^2 - 1\right]^2 + \left(\frac{2\beta f_n}{if_{step}}\right)^2}} \tag{2-17}$$

where

$a_{p,i}/g$ = peak acceleration for harmonic i as a fraction of the acceleration due to gravity

f_n = fundamental natural frequency, Hz

f_{step} = step frequency, Hz

i = harmonic number

w_p = unit weight of rhythmic activity with participants distributed over the entire bay, psf

w_t = unit weight supported including dead loads and superimposed dead load with occupants and participants distributed over the entire bay, psf

α_i = dynamic coefficient for the ith harmonic (Table 2-4)

β = damping ratio, normally taken as 0.06

With the 1.3 factor, Equation 2-17 applies only to simple beam and bay conditions, not cantilever beams or cantilever beams with backspans.

The peak acceleration ratio, a_p/g, accounting for all harmonics, is estimated from the 1.5 power combination rule (Allen, 1990b). The evaluation criterion is then,

$$\frac{a_p}{g} = \frac{\left(\sum a_{p,i}^{1.5}\right)^{1/1.5}}{g} \le \frac{a_o}{g} \tag{2-18}$$

where a_o/g is the tolerance limit acceleration ratio from Chapter 5 for commonly affected occupancies such as diners being affected by dancers. Experience shows, however, that many problems with building vibrations due to rhythmic exercises concern more sensitive occupancies in the building, especially for those located near an exercising area. For these occupancies, the maximum acceleration ratio calculated for the exercise floor should be adjusted in accordance with the vibration mode shape for the structural system (see Section 6.1.2), before comparing it to the acceleration limit for the sensitive occupancy from Figure 2-1.

Guidance on the estimation of parameters and examples of the application of Equation 2-18 are given in Chapter 5.

Chapter 3
Natural Frequency of Steel-Framed Floor Systems

The most important parameter for the vibration serviceability design and evaluation of floor framing systems is usually natural frequency. This chapter gives guidance for estimating the natural frequency of steel beam– and steel joist–supported floors and pedestrian bridges, including the effects of continuity. Example calculations are found in Chapter 4.

3.1 FUNDAMENTAL RELATIONSHIPS

Steel-framed floors are typically two-way systems that may have several vibration modes with closely spaced frequencies that are difficult to predict with certainty using any current method. Factors that are difficult to incorporate are composite action, boundary and discontinuity conditions, partitions, other nonstructural components, and member end stiffnesses. However, the fundamental mode often provides the majority of the total response and the fundamental natural frequency can be computed fairly accurately using the following closed-form solutions.

The floor is assumed to consist of a concrete slab (or deck) supported on steel beams or joists that are supported by walls, steel girders or joist girders between columns. The natural frequency, f_n, of the fundamental mode is estimated by considering a "beam or joist panel" mode and a "girder panel" mode separately and then combining them.

Beam or joist and girder panel mode natural frequencies can be estimated using the equation for fundamental natural frequency, f_n, of a simply supported beam with uniform mass:

$$f_n = \frac{\pi}{2}\left(\frac{gE_sI_t}{wL^4}\right)^{1/2} \tag{3-1}$$

where

E_s = modulus of elasticity of steel = 29,000 ksi

I_t = transformed moment of inertia; effective transformed moment of inertia if shear deformations are included; reduced transformed moment of inertia to account for joist seat flexibility, in.^{4}

L = member span, in.

g = acceleration of gravity = 386 in./s^2

w = uniformly distributed weight per unit length (actual, not design, dead and live loads) supported by the member, kip/in.

The combined mode or system frequency can be estimated using the Dunkerley relationship:

$$\frac{1}{f_n^2} = \frac{1}{f_j^2} + \frac{1}{f_g^2} \tag{3-2}$$

where

f_g = girder panel mode frequency, Hz

f_j = beam or joist panel mode frequency, Hz

Equation 3-1 can be rewritten as

$$f_n = 0.18\sqrt{\frac{g}{\Delta}} \tag{3-3}$$

where

Δ = midspan deflection of the member relative to its supports due to the supported weight, in.

$= \dfrac{5wL^4}{384E_sI_t}$

Sometimes, as described later in Sections 3.4 and 3.5, shear deformations must also be included in determining the deflection, Δ, which is accomplished by using an effective transformed moment of inertia.

For the combined mode, if both the beam or joist and girder are assumed simply supported, the Dunkerley relationship can be rewritten as

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g}} \tag{3-4}$$

where

Δ_g = girder midspan deflection due to the weight supported, in.

Δ_j = beam or joist and girder midspan deflection due to the weight supported, in.

Tall buildings can have vertical column frequencies low enough to create serious resonance problems with rhythmic activity, especially aerobics. For these cases, Equation 3-4 is modified to include the column effect:

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g + \Delta_c}} \tag{3-5}$$

where

Δ_c = axial shortening of the column or wall due to the weight supported, in.

Equations 3-1 and 3-3 are based on the assumption of a simply supported, uniformly loaded member. Joists, beams and girders are usually uniformly loaded, or nearly so, with the exception of girders that support joists or beams at midspan only. For such girders, the calculated girder deflection used in Equation 3-3 should be multiplied by $4/\pi \approx 1.3$ to take into account the difference between the frequency for a simply supported beam with distributed mass and that with a concentrated mass at midspan.

The recommended tolerance evaluation criterion for walking in Chapter 4 was calibrated using Equations 3-1 and 3-4. Equation 3-5 is recommended in Chapter 5 only for evaluating vibrations due to aerobic activities in the lower floors of tall buildings. The tolerance evaluation criteria in Chapter 6 were calibrated using $f_n = \min(f_j, f_g)$, where f_j and f_g are determined from Equation 3-1 or 3-2.

3.2 COMPOSITE ACTION

In calculating the fundamental natural frequency using the relationships in Section 3.1, the transformed moment of inertia is to be used if the slab (or deck) is attached to the supporting member. This assumption is to be applied even if structural shear connectors are not used because the shear forces caused by human activities at the slab/member interface can be resisted by deck-to-member spot welds or by friction between the concrete and the supporting member even when a deck is not present.

If the supporting member is separated from the slab (e.g., the case of overhanging beams that pass over a supporting girder or open-web joist seats), full composite behavior should not be assumed. See Section 3.5 for estimating the effects of joist seats. (If needed, the fundamental natural frequency of the girder can be increased by providing shear connection between the slab and girder flange—see Section 8.3.)

To account for the greater stiffness of concrete on metal deck under dynamic loading, as compared to static loading, it is recommended that the concrete modulus of elasticity be taken equal to 1.35 times that specified in current structural standards for calculation of the transformed moment of inertia. (It is noted that lower values of the dynamic multiplier are found in the literature, especially for high-strength concretes. However, all of the criteria recommended in this Design Guide were calibrated using the 1.35 value.) Also, for determining the transformed moment of inertia of typical beams or joists and girders, it is recommended that the effective width of the concrete slab be taken as the member spacing, but not more than 0.4 times the member span. For edge or spandrel members, the effective slab width is to be taken as one-half the member spacing, but not more than 0.2 times the member span, plus the projection of the free edge of the slab beyond the member centerline. If the concrete side of the member is in compression, the concrete can be assumed to be solid and uncracked. Based on limited experimental data, it is recommended that the concrete be assumed to be uncracked for cantilever members as well.

3.3 SUPERIMPOSED LOADS FOR VIBRATION ANALYSES

The response of a floor due to human activity is typically greatest when the floor is lightly loaded. Therefore, strength design dead and live loads should not be used for vibration analysis. Instead, estimates of actual or expected day-to-day dead and live loads should be used.

To estimate actual dead loads, it is recommended that for normal mechanical and ceiling installations, 4 psf be added to the structural system weight. For special cases, this value should be adjusted up or down as necessary.

Recommended live loads for vibration analysis are shown in Table 3-1. A "paper office" is one with heavy desks, file cabinets, bookcases and demountable partitions. An "electronic office" is one with widely spaced workstations, few desks and few demountable partitions. An "assembly area" is a meeting room or similar space with little furniture other than chairs. Vibration complaints for assembly areas, including schools and churches, usually occur when there are few people on the floor, e.g., a teacher and one or two students, not the entire class, or early arrivers for church services. The same is true for shopping malls and pedestrian bridges. Therefore, for these situations, a live load of 0 psf is recommended for vibration response prediction. The live load recommendations for paper offices and residences in Table 3-1 are the same as for the first edition of this Design Guide; electronic offices live load recommendations are in "Floor Vibrations and the Electronic Office" (Murray, 1998).

3.4 DEFLECTION DUE TO SHEAR IN BEAMS AND TRUSSES

Shear may contribute substantially to the deflection of the member. Two types of shear may occur: direct shear due to shear strain in the web of a beam or girder or due to length changes of the web members of a truss or indirect shear in trusses as a result of eccentricity of member forces through joints. For wide-flange members, the shear deflection is simply equal to the accumulated shear strain in the web from the support to midspan. For rolled shapes, shear deflection is usually small relative to flexural deflection and can be neglected. For simply-supported trusses with span-to-depth ratios greater than approximately 12, the shear deformation effect can usually be taken into account using Equation 3-6. For other cases, finite element analysis should be used. The effective transformed moment of inertia of the truss, accounting for shear deformation, I_e, is

$$I_e = \frac{I_{comp}}{1 + 0.15 I_{comp} / I_{chords}} \tag{3-6}$$

Table 3-1. Recommended Superimposed Live Loads for Walking Vibration Analyses	
Occupancy	**Recommended Live Load, psf**
Paper office	11
Electronic office	6–8
Residence	6
Assembly area	0
Shopping mall	0

where

I_{chords} = moment of inertia of the chord areas alone, in.^{4}

I_{comp} = fully composite transformed moment inertia of slab and chord areas, in.^{4}

3.5 SPECIAL CONSIDERATIONS FOR OPEN WEB JOISTS AND JOIST GIRDERS

Web shear deformations and eccentricity at joist and joist-girder panel points cause member flexibility to be more than that computed assuming flexural deformations alone. From research by Band and Murray (1996), the effective moment of inertia, I_e, which accounts for both effects, can be estimated using

$$I_e = \frac{1}{\dfrac{\gamma}{I_{chords}} + \dfrac{1}{I_{comp}}} \tag{3-7}$$

where

I_{chords} = moment of inertia of the chord areas alone, in.^{4}

I_{comp} = fully composite transformed moment of inertia of the slab and chord areas, in.^{4}

I_e = effective moment of inertia of the joist or joist girder accounting for shear deformations and joint eccentricities, in.^{4}

and

$$\gamma = \frac{1}{C_r} - 1 \tag{3-8}$$

For joists or joist girders with single- or double-angle web members with $6 \le L/D$

$$C_r = 0.90\left(1 - e^{-0.28(L/D)}\right)^{2.8} \le 0.9 \tag{3-9a}$$

and for joists with continuous round rod web members with $10 \le L/D$

$$C_r = 0.721 + 0.00725\left(\frac{L}{D}\right) \le 0.9 \tag{3-9b}$$

with

D = nominal depth of joist or joist girder, in.

L = joist or joist-girder span, in.

It was also found that joist seats are not sufficiently stiff to justify the full transformed moment of inertia assumption for joist girders or girders supporting standard joists. Based on research by Band and Murray (1996), the effective moment of inertia of joist girders supporting standard joist seats is estimated using

$$I_g = C_r I_{chords} + \frac{I_e - C_r I_{chords}}{4} \tag{3-10}$$

where C_r is from Equation 3-9a or 3-9b and I_e is the effective composite moment of inertia from Equation 3-7.

The effective moment of inertia of hot-rolled or built-up girders supporting standard joist seats is estimated using

$$I_g = I_x + \frac{I_{comp} - I_x}{4} \tag{3-11}$$

where

I_{comp} = fully composite moment of inertia of the slab and girder areas, in.^{4}

I_x = moment of inertia of the girder, in.^{4}

The Steel Joist Institute Technical Digest 5, *Vibration Analysis of Steel Joist-Concrete Floor Systems* (Murray and Davis, 2015) includes more information on the analysis of joist and joist-girder supported floors.

Chapter 4
Design for Walking Excitation

The design criterion for walking excitations recommended in Section 2.2 is based on the dynamic response of steel beam and joist-supported floor systems to walking forces. The criterion can be used to evaluate concrete/steel-framed structural systems supporting offices, residences, churches, schools, and other quiet spaces, as well as shopping malls and pedestrian bridges and, in a modified form, monumental stairs. The following sections demonstrate the application of the criterion and show example calculations.

4.1 RECOMMENDED EVALUATION CRITERION FOR LOW-FREQUENCY BUILDING FLOORS

The following sections describe the recommended human comfort criterion for the evaluation of floor vibration due to walking on low-frequency floors, ($f_n \le 9$ Hz). Such floors are subject to resonant build-up as described in Chapter 1. Occupant complaints of objectionable vibration of high-frequency floors, ($f_n > 9$ Hz,) are rare and are not considered in this chapter. If an evaluation of a high-frequency floor is needed, the criteria in Section 2.2 can be used with calculations similar to those shown in Example 6.2. Procedures for evaluating floors supporting sensitive equipment are in Chapter 6. Systems with geometry or other features outside the scope of this chapter can be evaluated using finite element analysis methods in Chapter 7.

4.1.1 Criterion

The recommended criterion for low-frequency building floors states that the floor system is satisfactory if the peak acceleration, a_p, due to walking excitation as a fraction of the acceleration of gravity, g, determined from

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \tag{4-1}$$

does not exceed the tolerance acceleration limit, a_o/g, for the appropriate occupancy,

where
- P_o = amplitude of the driving force, 65 lb
- W = effective weight supported by the beam or joist panel, girder panel, or combined panel, as applicable, lb
- f_n = fundamental natural frequency of a beam or joist panel, a girder panel, or a combined panel, as applicable, Hz
- β = damping ratio

Note that the constant force, P_o, does not represent the weight of the walker; it is the amplitude of the driving force, as explained in Section 2.2. For typical quiet spaces supported by two-way systems (beams or joists and girders), it is recommended that the reduction factor, R (see Equation 2-3), be taken as 0.5 to account for the walker and affected occupant (sensor) not being at the same location, resulting in a P_o value of 65 lb. The reduction factor, R, and therefore, P_o, can be increased or decreased to meet particular needs of a specific design.

Recommended acceleration tolerance limits, a_o/g, are found in Table 4-1. These limits are slightly conservative for natural frequencies between 3 Hz and 4 Hz and 8 Hz and 9 Hz compared to values from Figure 2-1 but are recommended for design simplicity. The recommended limits are the same as those in the first edition of this Design Guide. Floor systems with fundamental frequencies less than 3 Hz should generally be avoided because they are liable to be subjected to "rogue or vandal jumping". If $f_n < 3$ Hz, the system should be evaluated using criteria in Chapter 5.

The following provides guidance for estimating required properties for application of the recommended criterion to building floors.

4.1.2 Estimation of Required Parameters

The fundamental natural frequency, f_n, is determined as described in Chapter 3. The effective panel weight, W, and damping ratio, β, are estimated as follows.

Effective Panel Weight, W

The effective panel weight is estimated by determining the effective panel weights for the beam or joist panel and girder panel modes separately and then combining them in proportion to their flexibilities. The effective panel weights, W, for the beam or joist and girder panel modes are estimated from

$$W = wBL \tag{4-2}$$

where
- B = effective panel width, ft
- L = member span, ft
- w = supported weight per unit area, psf

For the beam or joist panel mode, the effective width is

$$B_j = C_j \left(\frac{D_s}{D_j}\right)^{1/4} L_j \le \left(\frac{2}{3}\right) \textit{floor width} \tag{4-3}$$

Table 4-1. Recommended Tolerance Limits for Building Floors	
Occupancy	**Acceleration Limit $a_o/g \times 100\%$**
Offices, residences, churches, schools and quiet areas	0.5%
Shopping malls	1.5%

where

C_j = 2.0 for joists or beams in most areas
= 1.0 for joists or beams parallel to a free edge (edge of balcony, mezzanine, or building edge if cladding is not connected)

E_c = modulus of elasticity of concrete = $w^{1.5}\sqrt{f_c'}$, ksi

E_s = modulus of elasticity of steel = 29,000 ksi

D_j = joist or beam transformed moment of inertia per unit width, in.^{4}/ft

$$= I_j/S \tag{4-3a}$$

D_s = slab transformed moment of inertia per unit width, in.^{4}/ft, can be taken from a deck manufacturer's catalog, or is approximately

$$= \frac{12d_e^3}{12n} \tag{4-3b}$$

I_j = transformed or effective moment of inertia of the beam or joist, in.^{4}

L_j = joist or beam span, ft

S = joist or beam spacing, ft

d_e = effective depth of the concrete slab, taken as the depth of the concrete above the deck plus one-half the depth of the deck, in.

n = dynamic modular ratio

$$= E_s/1.35E_c \tag{4-3c}$$

Floor width is the distance perpendicular to the joist or girder span of the beams or joists in the bay under consideration over which the structural framing (beam or joist and girder size, spacing, length, etc.) is identical or nearly identical in adjacent bays.

For the girder panel mode, the effective width, except for edge girders, is

$$B_g = C_g\left(\frac{D_j}{D_g}\right)^{1/4} L_g \le \left(\frac{2}{3}\right) \textit{floor length} \tag{4-4}$$

where

C_g = 1.6 for girders supporting joists connected to the girder flange with joist seats
= 1.8 for girders supporting beams connected to the girder web

D_g = girder transformed moment of inertia per unit width, in.^{4}/ft
= I_g divided by the average span of the supported beams or joists

L_g = girder span, ft

Floor length is the distance perpendicular to the span of the girders in the bay under consideration over which the structural framing (beam or joist and girder size, spacing, length, etc.) is identical or nearly identical in adjacent bays.

For edge girders, the effective width is two-thirds of the supported beam or joist span.

Where beams, joists or girders are continuous over their supports and an adjacent span is greater than 0.7 times the span under consideration, the effective panel weight, W_j or W_g, can be increased by 50%. This liberalization also applies to rolled sections shear-connected to girder webs but not to joists connected only at their top chord. When joist bottom-chord extensions are installed and connected before concrete is placed, the effective weight of the joist mode can be increased by 30% (Avci, 2014). Because continuity effects are not generally realized when girders frame directly into columns, either shear or moment connected, this increase does not apply to such girders. If the girder passes over a column top, the increase is applicable.

For the combined mode, the effective panel weight is estimated using

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \tag{4-5}$$

where

W_g = effective panel weights from Equation 4-2 for the girder panels, lb

W_j = effective panel weights from Equation 4-2 for the beam or joist, lb

Δ_g = midspan deflections of the girder due to the weight supported by the member, in.

Δ_j = midspan deflections of the beam or joist due to the weight supported by the member, in.

Composite action with the concrete deck is typically assumed when calculating Δ_j and Δ_g, provided there is sufficient shear connection between the slab/deck and the member. See Sections 3.2, 3.4 and 3.5 for more details.

If the girder span, L_g, is less than the joist panel width, B_j, the combined mode is restricted and the system is effectively stiffened. This can be accounted for by reducing the deflection, Δ_g, used in Equation 4-5 to a reduced deflection, Δ_g',

$$\Delta_g' = \frac{L_g}{B_j}\left(\Delta_g\right) \tag{4-6}$$

Table 4-2. Recommended Component Damping Values for Use in Equation 4-1	
Component	Ratio of Actual Damping-to-Critical Damping, β_i
Structural system	0.01
Ceiling and ductwork	0.01
Electronic office fit-out	0.005
Paper office fit-out	0.01
Churches, schools and malls	0.0
Full-height dry wall partitions in bay	0.02 to 0.05*
*Depending on the number of partitions in the bay and their location; nearer the center of the bay provides more damping.	

where L_g/B_j is taken as not less than 0.5 nor greater than 1.0 for calculation purposes, i.e., $0.5 \le L_g/B_j \le 1.0$.

If the beam or joist span is less than one-half the girder span, the beam or joist panel mode and the combined mode should be checked separately.

Damping

The damping ratio, β, can be estimated using the component values shown in Table 4-2, noting that damping is cumulative. For example, a floor with ceiling and ductwork supporting an electronic office area has $\beta = \Sigma\beta_i = 0.01 + 0.01 + 0.005 = 0.025$, or 2.5% of critical damping.

4.1.3 Design Considerations

Open Web Joists

As shown in Figure 4-1, an open-web joist is typically supported at the ends by a seat on the girder flange and the bottom chord is not connected to the girders. This support detail provides much less flexural continuity than shear connected beams, reducing both the bending stiffness of the girder panel and the participation of the mass of adjacent bays in resisting walker-induced vibration. These effects are accounted for as follows:

1. The reduced bending stiffness requires that the coefficient 1.8 in Equation 4-4 be reduced to 1.6 when joist seats are present.
2. The nonparticipation of mass in adjacent bays means that an increase in effective joist panel weight should not be considered; that is, the 50% increase in panel weight, as recommended for shear-connected beam-to-girder connections should not be used. If bottom-chord extensions are installed before the concrete slab is placed, a 30% increase in panel weight can be used (Avci, 2014).

Also, the separation of the girder from the concrete slab results in partial composite action, and the moment of inertia of girders supporting joist seats should therefore be determined using the procedure in Section 3.5.

More information on joist-supported floors is found in the Steel Joist Institute Technical Digest 5, *Vibration Analysis of Steel Joist-Concrete Floor Systems* (Murray and Davis, 2015).

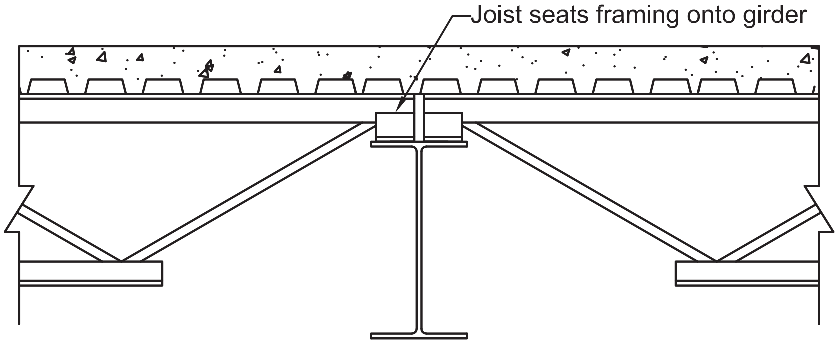


Fig. 4-1. Typical joist support.

Unequal Beam or Joist Spans

For the common situation where the girder stiffnesses or effective girder panel weights in a bay are different, the following modifications to the basic design procedure are necessary:

1. The combined mode frequency should be determined using the girder with the greater Δ_g or lower f_g.
2. The effective girder panel width should be determined using the average span length of the beams or joists supported by the girder with lower natural frequency, i.e., the average beam or joist span length is substituted for L_j when determining D_g.
3. In some instances, calculations may be required for both girders to determine the critical case.

Interior and Exterior Floor Edges

Interior floor edges, as in mezzanine areas or atria, or exterior floor edges that are not connected to exterior cladding, require special consideration because of the reduced effective mass due to the free edge. Where the edge member is a joist or beam, a practical solution is to stiffen the edge by adding another joist or beam or by choosing an edge beam with moment of inertia 50% greater than for the interior beams. If the edge joist or beam is not stiffened, the effective panel weight, W_j, should be computed with the coefficient C_j in Equation 4-3 taken as 1.0. Where the edge member is a girder, the effective panel weight, W_g, should be computed with the girder panel width, B_g, taken as two-thirds of the supported beam or joist span. See Examples 4.3 and 4.4.

Floor Width and Floor Length

Floor width is the distance perpendicular to the span of the joists over which the structural framing (beam or joist and girder size, spacing, length, etc.) is identical or nearly identical within adjacent bays and represents to some degree the width of the mode shape associated with the joist panel. *Floor length* is the distance perpendicular to the span of the girders over which the structural framing (beam or joist and girder size, spacing, length, etc.) is identical or nearly identical within adjacent bays and represents to some degree the length of the mode shape associated with the girder panel.

For example, the *floor width* and *floor length* for Bays A, B and C for the framing in Figure 4-2 are as shown in Table 4-3.

Vibration Transmission

Occasionally, a floor system will be judged particularly objectionable because of vibration transmission transverse to the supporting joists or beams. In these situations, when the floor is impacted at one location, there is a perception that a "wave" moves from the impact location in a direction transverse to the supporting joists or beams. The phenomenon is described in more detail in Section 8.3. The recommended criterion does not address this phenomenon, but

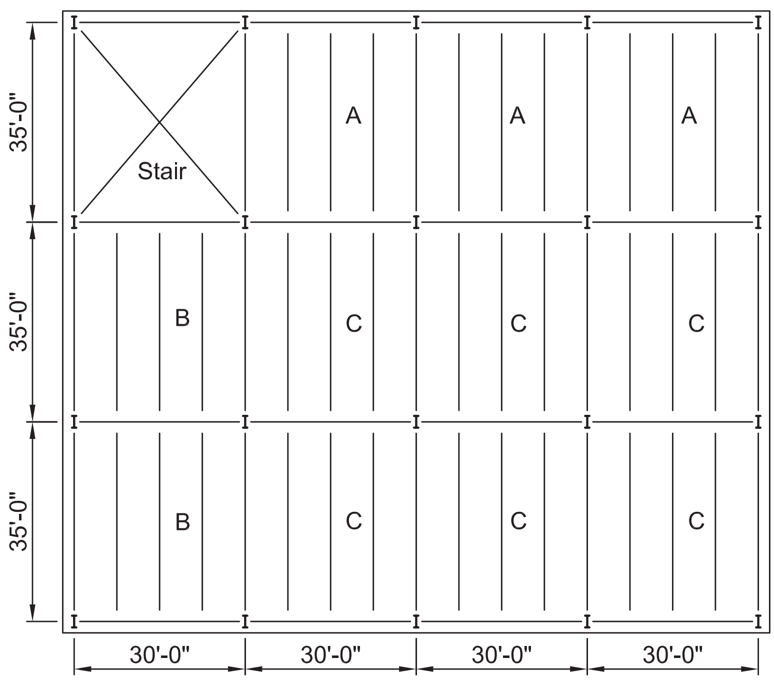


Fig. 4-2. Floor width and floor length example framing.

Table 4-3. *Floor Lengths* and *Floor Widths* for Figure 4-2 Framing		
Bay	***Floor Width*, ft**	***Floor Length*, ft**
A	90	105
B	120	70
C	120	105

Table 4-4. Recommended Tolerance Limits for Pedestrian Bridges	
Type	**Acceleration Limit $a_o/g \times 100\%$***
Indoor	1.5%
Outdoor	5.0%
*For standing pedestrians. Lower values may be appropriate if seating is provided.	

a small change in the structural system will usually eliminate the problem. If one beam or joist stiffness or spacing is changed periodically—say, by 50% in every third bay—the "wave" is interrupted at that location and floor motion is much less objectionable. Full-height partitions may achieve the same result.

Summary

Figure 4-3 is a summary of the procedure for assessing typical low-frequency building floors for walking vibrations.

4.2 RECOMMENDED EVALUATION CRITERIA FOR PEDESTRIAN BRIDGES

The following presents recommended criteria and analysis examples for indoor and outdoor pedestrian bridges.

The evaluation criterion for floors can also be used to determine the vertical vibration acceptance of pedestrian bridges supported by beams or joists and girders. Recommended tolerance acceleration limits are shown in Table 4-4. A reduction factor of 0.7 is recommended in Section 2.2 for establishing the driving force because pedestrian bridges are one-way systems, and the walker and the potentially affected sensor can be relatively close together. The resulting P_o value is 92 lb, assuming there is only one walker. Bachmann and Ammann (1987) have suggested that for marching by a group, the dynamic loading is the number of walkers, n, times that of a single walker, that is, nP_o. And, for a group of random walkers, it is $\sqrt{n}$ times that for a single walker, $\sqrt{n}P_o$.

The recommended damping ratio for pedestrian bridges is 0.01, assuming there is only bare structural framing. If a soffit or other element that increases damping exists, the ratio should be increased. The effective weight, W, is taken as the total weight of the bridge. The acceleration limit for outdoor footbridges should not be used for quiet areas like crossovers in hotel or office building atria. The maximum step frequency is 2.2 Hz, so the maximum lateral forcing frequency is 1.1 Hz. Synchronization of walking with lateral sway will not occur if the natural frequency of lateral vibration exceeds 1.1 Hz. Thus, it is recommended that the natural frequency of lateral vibration be not less than 1.3 Hz (AASHTO, 2009).

Designers of pedestrian bridges are cautioned to pay attention to the location of the concrete slab relative to the beam height. If the concrete slab is located between the beams (because of clearance considerations), the pedestrian bridge will vibrate at a much lower frequency and at larger amplitude than if the slab is located above the supporting members, because of the lower transformed moment of inertia.

4.3 RECOMMENDED EVALUATION CRITERIA FOR LINEAR MONUMENTAL STAIRS

Evaluation of linear monumental stairs for walking vibration tolerance consists of three checks (see Section 2.3): (1) that the vertical natural frequency of the stair is greater than 5 Hz, (2) that lateral natural frequency is greater than 2.5 Hz, and (3) that the vertical acceleration due to a descending individual or group is less than the relevant tolerance limit for people standing on the stairs. Recommended step frequencies for normal and rapid descents and acceleration tolerance limits for people standing on the stairs—not the walkers—are shown in Table 4-5. Because stair descent accelerations are always greater than ascent accelerations, only descents need to be considered in design.

The procedures recommended in the following can be used to analyze linear flights of stairs, such as shown in Figure 4-4(a). The procedures can also be adapted using engineering judgment for stairs, such as the one shown in Figure 4-4(b). The finite element method in Chapter 7 should be used for more complex slender stairs.

A. FLOOR SLAB

Determine uniformly distributed weight, total depth, deck height, and effective depth, d_e.

Calculate $n = E_s/(1.35E_c)$.

B. JOIST PANEL MODE

Calculate I_j (see Section 3.4 if trusses or Section 3.5 if open web joists).

Calculate w_j and $\Delta_j = \dfrac{5w_jL_j^4}{384E_sI_j}$.

Calculate $f_j = 0.18\sqrt{g/\Delta_j}$.

Determine D_s for slab and deck or estimate using $D_s = \left(12d_e^3\right)/12n$.

Calculate $D_j = I_j/S$.

Calculate $B_j = C_j\left(D_s/D_j\right)^{1/4}L_j \le$ (⅔) (*floor width*).

$C_j = 2.0$ for interior panels; 1.0 for edge panels.

Calculate $W_j = w_jB_jL_j$ ($\times$ 1.5 if continuous or web connected or 1.3 if joist bottom chords are extended, and an adjacent beam or girder span is greater than 0.7 times the joist or beam span of the bay).

C. GIRDER PANEL MODE

For each girder:

Calculate I_g (Section 3.4 if a truss; Section 3.5 if a joist girder; Section 3.5 if open web joists are supported).

Calculate w_g and $\Delta_g = \dfrac{5w_gL_g^4}{384E_sI_g}$ with correction if only one beam is supported at midspan (see Section 3.1).

Calculate $f_g = 0.18\sqrt{g/\Delta_g}$ and $D_g = I_g/L_j$.

Use average of supported joist span lengths, if different, for L_j.

If girder frequencies are different, base remainder of calculations on the girder with lower frequency.

For interior panel, calculate

$B_g = C_g\left(D_j/D_g\right)^{1/4}L_j \le$ (⅔) (*floor length*)

$C_g = 1.8$ if shear connected; 1.6 if not.

For edge panel, calculate $B_g = \left(\dfrac{2}{3}\right)L_j$.

Calculate $W_g = w_gB_gL_g$($\times$ 1.5 if girder is continuous over the top of supporting columns and an adjacent girder span is greater than 0.7 times the girder span in the bay).

D. COMBINED PANEL MODE

Calculate $f_n = 0.18\sqrt{g/\left(\Delta_j + \Delta_g\right)}$.

If $B_j > L_g$, reduce Δ_g by $L_g/B_j \ge 0.5$ (Equation 4-6).

Calculate $W = \dfrac{\Delta_j}{\Delta_j + \Delta_g}W_j + \dfrac{\Delta_g}{\Delta_j + \Delta_g}W_g$.

Estimate β using values from Table 4-2.

Calculate $\dfrac{a_p}{g} = \dfrac{P_o\exp\left(-0.35f_n\right)}{\beta W}$ where $P_o = 65$ lb or as modified for a particular design (see Section 4.1.1).

Compare $\dfrac{a_p}{g}$ to $\dfrac{a_o}{g}$ from Table 4-1.

Fig. 4-3. Floor evaluation calculation procedure.

Table 4-5. Vertical Acceleration Tolerance Limits and Parameters				
Step Frequency, Hz	**Acceleration Tolerance Limit, a_o, %g**	**Calibration Factor, R**	**Walking Load Parameter, γ**	**Remarks**
≤ 2.5	1.7	0.7	0.29	Normal descents
2.5–4.0	3.0	0.5 if $f_n < 8$ Hz 0.7 if $f_n > 8$ Hz	0.19	Rapidly descending individual—not perceptible
2.5–4.0	4.5	0.5 if $f_n < 8$ Hz 0.7 if $f_n > 8$ Hz	0.19	Rapidly descending individual—perceptible; rapidly descending group

Natural Frequencies

Stair vertical or lateral natural frequency can be determined using Equation 3-1 with slightly different definitions as follows:

$$f_n = \frac{\pi}{2}\left(\frac{gE_sI_t}{W_sL_s^3}\right)^{1/2} \tag{4-7}$$

where

E_sI_t = stringer vertical flexural stiffness, including stringers and any other elements that provide stiffness; stair lateral flexural stiffness, lb-in.^{2}

L_s = stringer length measured along the diagonal between supports, in.

W_s = weight of stair, lb

f_n = fundamental natural frequency, Hz

g = acceleration of gravity = 386 in./s^2

If the stair is supported on girders, the vertical combined mode or system frequency can be estimated using the Dunkerley relationship as shown in Equation 3-2.

Acceptance Criterion

The acceleration acceptance criterion, Inequality 4-8, for vertical vibration of linear stairs is similar to that for floors but somewhat more complex as explained in Section 2.3. The criterion states that the stair is satisfactory if the peak acceleration, a_p, due to a stair descent as a fraction of the acceleration of gravity, g, does not exceed the acceleration tolerance limit, a_o in %/g, from Table 4-5:

$$\frac{a_p}{g} = 0.62e^{-\gamma f_n}\frac{RQ\cos^2\theta}{\beta W_s}\phi_W\phi_R\left(1-e^{-100\beta}\right) \le \frac{a_o}{g} \tag{4-8}$$

where

Q = assumed bodyweight = 168 lb

R = calibration factor (see Table 4-5)

W_s = weight of stair, lb

β = damping ratio

ϕ_W = unity normalized mode shape value at the excitation (walker)

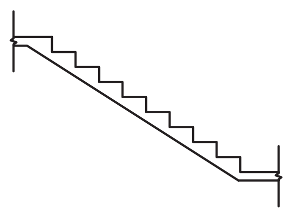
ϕ_R = unity normalized mode shape value at the response (potentially affected observer) location

θ = stair inclination from horizontal, measured with respect to support points, degrees

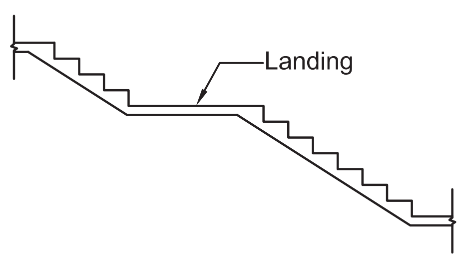
γ = 0.29 for normal descents
= 0.19 for rapid descents

Because the vertical natural frequency is greater than 5 Hz and maximum assumed step frequency for normal descents from Table 4-5 is 2.5 Hz, the harmonic number, h, is greater than 2 and the calibration factor, R, is 0.7 as specified in Section 2.3. Similarly, if the vertical frequency is less than 8 Hz, the harmonic number is 2 for rapid descents and R = 0.5. If the natural frequency is greater than 8 Hz, the harmonic number is 3 or greater and R = 0.7.

Engineering judgment is required when estimating the damping ratio. Davis and Murray (2009) reported a damping ratio of 0.01 for a stair with no nonstructural components, treads that are isolated from each other, and guardrails that are connected without the potential for frictional interfaces.



(a) Linear stair



(b) Linear stair with intermediate landing

Fig. 4-4. Linear stairs.

Davis and Avci (2015) reported a damping ratio of 0.038 for a stair with drywall soffit, treads and risers with frictional interfaces, and guardrails that are connected to the stringers with frictional interfaces. For such stairs, it is recommended that β be assumed to be between 0.03 and 0.04.

The fundamental mode shape is a half sine wave over the stringer length, L_s, so the mode shape amplitudes at the walker location, x_W, and response (observer) location, x_R, respectively, are:

$$\phi_W = \sin\left(\frac{\pi x_W}{L_s}\right) \tag{4-9}$$

$$\phi_R = \sin\left(\frac{\pi x_R}{L_s}\right) \tag{4-10}$$

where

x_R = distance from end of stringer to response location, measured on the diagonal, in.

x_W = distance from end of stringer to walker excitation force location, measured on the diagonal, in.

The walker location, x_W, must be set using engineering judgment. Resonant build-up durations are highly variable, in the range of five to ten steps long, thus an eight-step resonant build-up is recommended for design. If a walker can achieve an eight-step build-up near midspan, then $\phi_R = 1.0$. If an intermediate landing exists at midspan, an eight-step series of stair treads centered at midspan is not possible; therefore, x_W is at the center of the eight-step series of stair treads closest to midspan. It is conservative to take ϕ_W and ϕ_R equal to 1.0.

Recommended Evaluation Procedures

The vertical and lateral natural frequencies should be greater than 5 Hz and 2.5 Hz, respectively.

It is recommended that the acceleration response due to an individual performing a normal speed descent, i.e., one with a step frequency below about 2.5 Hz, be evaluated first. For this evaluation, the predicted acceleration from Equation 4-8 is compared to the tolerance limit from Table 4-5 for step frequencies not exceeding 2.5 Hz, i.e., 1.7%g. If the tolerance limit is exceeded, redesign is necessary.

Next, the acceleration response due to an individual performing a rapid descent, i.e., one with a step frequency between 2.5 Hz and 4.0 Hz should be evaluated. For this evaluation, the predicted acceleration from Equation 4-8 is compared to the tolerance limit from Table 4-5, i.e., 3.0%g, if perceptible vibrations must be prevented, or 4.5%g otherwise. If the tolerance limit is exceeded, redesign is necessary.

If fast moving groups are likely, then the acceleration response is determined by amplifying the acceleration due to a rapidly moving individual by a factor of three. If the stair is wide enough to accommodate stationary people, then the predicted acceleration is compared to the recommended acceleration tolerance limit in Table 4-5, 4.5%g. This limit may be very difficult to satisfy for a slender monumental stair, and a much stiffer or heavier stair will often be required. If only walkers are considered—e.g., there is inadequate room for stationary people during a group descent—there is no recommended acceleration tolerance limit.

4.4 DESIGN EXAMPLES

Table 4-6 identifies the intent of each of the following examples.

Example 4.1—Typical Exterior Bay of an Office Building with Hot-Rolled Framing

Given:

The hot-rolled framing system for a typical exterior bay shown in Figure 4-5 is to be evaluated for walking vibration. The structural system supports "paper office" build-out with an assumed live load of 11 psf and there is ceiling and mechanical equipment below with an assumed superimposed dead load of 4 psf. The spandrel girder is supported by the exterior cladding for vibration analysis purposes. The building is five bays in the E-W direction and three bays in the N-S direction. The beams are connected to the girder webs. The slab is 5¼-in. total depth, lightweight concrete (w_c = 110 pcf, f'_c = 4 ksi), on 2-in.-deep deck. The assumed selfweight of the deck is 2 psf. The beam and girders are ASTM A992 material.

Table 4-6. Summary of Design Examples	
Example	Description
4.1	Typical exterior bay of an office building with hot-rolled framing
4.2	Typical interior bay of an office building with open web joist/hot-rolled girder framing
4.3	Mezzanine with beam edge member
4.4	Mezzanine with girder edge member
4.5	Pedestrian bridge—walking and running
4.6	Linear stair—individual and group loadings

Solution:

From the AISC *Steel Construction Manual* (AISC, 2011) Table 2-4, hereafter referred to as the AISC *Manual*, the material properties are as follows:

Beam and Girder
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beams
W18×35
$A = 10.3$ in.^{2}
$I_x = 510$ in.^{4}
$d = 17.7$ in.

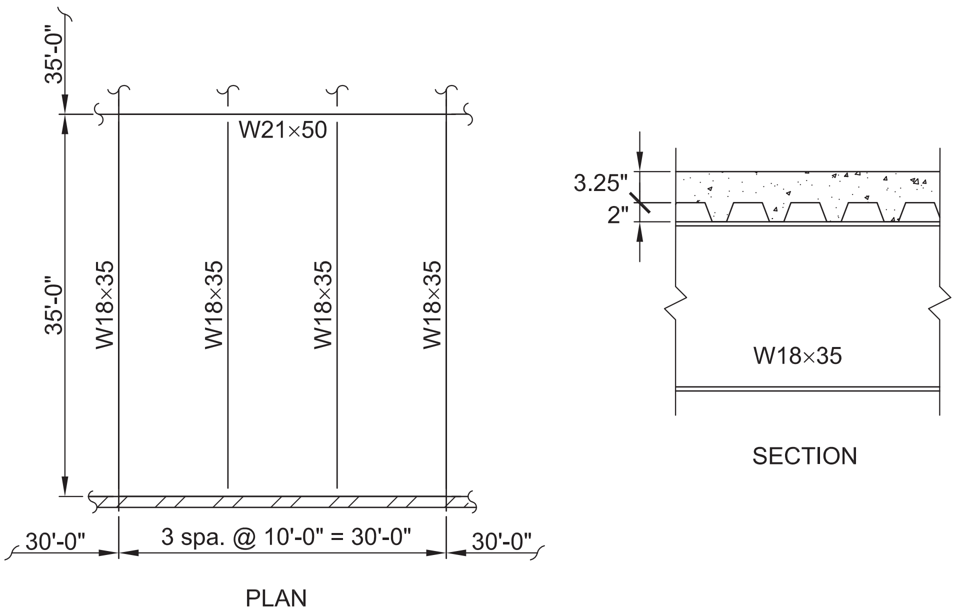


Fig. 4-5. Exterior bay floor framing details for Example 4.1.

Girder
W21×50
A = 14.7 in.^{2}
I_x = 984 in.^{4}
d = 20.8 in.

The geometric layout of the floor is calculated as follows:

$$\begin{aligned} \textit{Floor width} &= (5 \text{ bays})(30.0 \text{ ft}) \\ &= 150 \text{ ft} \\ \textit{Floor length} &= (3 \text{ bays})(35.0 \text{ ft}) \\ &= 105 \text{ ft} \end{aligned}$$

From the recommended values given in Table 4-2, the estimated damping ratio is determined as follows:

$$\begin{aligned} \beta &= 0.01 \text{ (structural system)} + 0.01 \text{ (ceiling and ductwork)} + 0.01 \text{ (paper office fit-out)} \\ &= 0.03 \end{aligned}$$

The properties of the deck are determined as follows:

$$\begin{aligned} E_c &= w^{1.5}\sqrt{f_c'} \\ &= (110 \text{ pcf})^{1.5}\sqrt{4 \text{ ksi}} \\ &= 2{,}310 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{Slab} + \text{deck weight} &= \frac{\left(3.25 \text{ in.} + \dfrac{2.00 \text{ in.}}{2}\right)(110 \text{ pcf})}{12 \text{ in./ft}} + 2.00 \text{ psf} \\ &= 41.0 \text{ psf} \end{aligned}$$

Beam Transformed Moment of Inertia

Considering only the concrete above the steel form deck, and using a dynamic concrete modulus of elasticity of 1.35 E_c, the transformed moment of inertia of the fully composite beam is computed as follows:

$$n = \frac{E_s}{1.35E_c} \tag{4-3c}$$

$$\begin{aligned} &= \frac{29{,}000 \text{ ksi}}{1.35(2{,}310 \text{ ksi})} \\ &= 9.30 \end{aligned}$$

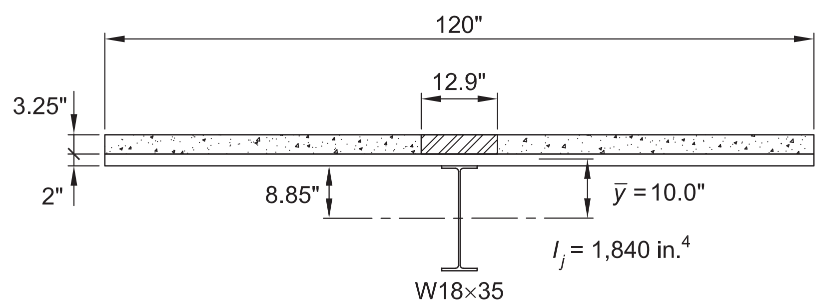


Fig. 4-6. Composite beam cross section for Example 4.1.

As shown in Figure 4-6, the effective concrete slab width is:

$$\begin{aligned}\min[0.4L_i, S] &= \min[0.4(35.0\text{ ft})(12\text{ in./ft}), 120\text{ in.}]\\ &= 120\text{ in.}\end{aligned}$$

The effective concrete slab depth is

$$5\tfrac{1}{4}\text{ in.} - 2.00\text{ in.} = 3.25\text{ in.}$$

The transformed concrete slab width is

$$120\text{ in.}/9.30 = 12.9\text{ in.}$$

The transformed concrete slab area is

$$(3.25\text{ in.})(12.9\text{ in.}) = 41.9\text{ in.}^2$$

The beam transformed moment of inertia is calculated as follows:

$$\begin{aligned}\bar{y} &= \frac{\left(41.9\text{ in.}^2\right)\left(\dfrac{17.7\text{ in.}}{2} + 2.00\text{ in.} + \dfrac{3.25\text{ in.}}{2}\right)}{41.9\text{ in.}^2 + 10.3\text{ in.}^2}\\ &= 10.0\text{ in. (above c.g. of beam)}\end{aligned}$$

$$\begin{aligned}I_j &= \frac{(12.9\text{ in.})(3.25\text{ in.})^3}{12} + \left(41.9\text{ in.}^2\right)\left(\frac{17.7\text{ in.}}{2} + 2.00\text{ in.} + \frac{3.25\text{ in.}}{2} - 10.0\text{ in.}\right)^2 + 510\text{ in.}^4 + \left(10.3\text{ in.}^2\right)(10.0\text{ in.})^2\\ &= 1{,}840\text{ in.}^4\end{aligned}$$

Beam Mode Properties

For each beam, the uniformly distributed loading is determined as follows, which includes 11 psf live load and 4 psf dead load for mechanical/ceiling.

$$\begin{aligned}w_j &= (10.0\text{ ft})(11.0\text{ psf} + 41.0\text{ psf} + 4.00\text{ psf}) + 35.0\text{ plf}\\ &= 595\text{ plf}\end{aligned}$$

The corresponding deflection is determined as follows:

$$\begin{aligned}\Delta_j &= \frac{5w_jL_j^4}{384E_sI_j}\\ &= \frac{5(595\text{ plf})(35.0\text{ ft})^4(1{,}728\text{ in.}^3/\text{ft}^3)}{384(29\times10^6\text{ psi})(1{,}840\text{ in.}^4)}\\ &= 0.376\text{ in.}\end{aligned}$$

The beam mode fundamental frequency from Equation 3-3 is

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} \tag{3-3}$$

$$\begin{aligned}&= 0.18\sqrt{\frac{386\text{ in./s}^2}{0.376\text{ in.}}}\\ &= 5.77\text{ Hz}\end{aligned}$$

Using an average concrete thickness of $d_e = 4.25$ in., the transformed slab moment of inertia per unit width in the slab span direction is

$$D_s = \frac{12d_e^3}{12n} \tag{4-3b}$$
$$= \frac{(12 \text{ in./ft})(4.25 \text{ in.})^3}{12(9.30)}$$
$$= 8.25 \text{ in.}^4/\text{ft}$$

The transformed moment of inertia per unit width in the beam span direction with beam spacing of 10 ft is

$$D_j = \frac{I_j}{S} \tag{4-3a}$$
$$= \frac{1{,}840 \text{ in.}^4}{10.0 \text{ ft}}$$
$$= 184 \text{ in.}^4/\text{ft}$$

The effective beam panel width from Equation 4-3 with $C_j = 2.0$, because it is a typical bay without a free edge, is

$$B_j = C_j \left(\frac{D_s}{D_j}\right)^{1/4} L_j \tag{4-3}$$
$$= 2.0\left(\frac{8.25 \text{ in.}^4/\text{ft}}{184 \text{ in.}^4/\text{ft}}\right)^{1/4}(35.0 \text{ ft})$$
$$= 32.2 \text{ ft}$$

Per Equation 4-3, the effective beam panel width must be less than two-thirds of the *floor width*. Because this is a typical exterior bay, the actual *floor width* is 5 times the girder span, 5(30.0 ft) = 150 ft. With ⅔(150 ft) = 100 ft > 32.2 ft, the effective beam panel width is 32.2 ft.

The weight of the beam panel is calculated from Equation 4-2. Because the adjacent beam span, 35 ft, is greater than 0.7 of the beam span and the beam is shear connected to the girder, the weight of the beam panel is adjusted by a factor of 1.5 to account for continuity as explained in Section 4.1.2:

$$W_j = 1.5\left(\frac{w_j}{S}\right)B_j L_j \tag{from Eq. 4-2}$$
$$= 1.5\left(\frac{595 \text{ plf}}{10.0 \text{ ft}}\right)(32.2 \text{ ft})(35.0 \text{ ft})$$
$$= 101{,}000 \text{ lb}$$

Girder Transformed Moment of Inertia

The effective concrete slab width is

$$\min[0.2L_g, 0.5L_{j,left}] + \min[0.2L_g, 0.5L_{j,right}] = \min[0.2(30.0 \text{ ft}), 0.5(35.0 \text{ ft})] + \min[0.2(30.0 \text{ ft}), 0.5(35.0 \text{ ft})]$$
$$= 6.00 \text{ ft} + 6.00 \text{ ft}$$
$$= 12.0 \text{ ft}$$

As shown in Figure 4-7, the transformed concrete slab width, using a dynamic concrete modulus of elasticity of $1.35E_c$, is

$$(12.0\text{ ft})(12\text{ in./ft})/9.30 = 15.5\text{ in.}$$

Assuming that the deck has a symmetrical profile, the effective width of the slab in the deck is taken as 72 in. The transformed concrete width of the deck is

$$72.0\text{ in.}/9.30 = 7.74\text{ in.}$$

The transformed concrete slab area is

$$(3.25\text{ in.})(15.5\text{ in.}) = 50.4\text{ in.}^2$$

The transformed concrete slab area in the deck is

$$(2.00\text{ in.})(7.74\text{ in.}) = 15.5\text{ in.}^2$$

The transformed moment of inertia is computed as follows:

$$\begin{aligned}
\bar{y} &= \frac{\left(50.4\text{ in.}^2\right)\left(\dfrac{20.8\text{ in.}}{2}+2.00\text{ in.}+\dfrac{3.25\text{ in.}}{2}\right)+\left(15.5\text{ in.}^2\right)\left(\dfrac{20.8\text{ in.}}{2}+\dfrac{2.00\text{ in.}}{2}\right)}{50.4\text{ in.}^2+15.5\text{ in.}^2+14.7\text{ in.}^2} \\
&= 11.0\text{ in. (above c.g. of girder)} \\
I_g &= \frac{(15.5\text{ in.})(3.25\text{ in.})^3}{12}+\left(50.4\text{ in.}^2\right)\left(\frac{20.8\text{ in.}}{2}+2.00\text{ in.}+\frac{3.25\text{ in.}}{2}-11.0\text{ in.}\right)^2+\frac{(7.74\text{ in.})(2.00\text{ in.})^3}{12} \\
&\quad +\left(15.5\text{in.}^2\right)\left(\frac{20.8\text{ in.}}{2}+\frac{2.00\text{ in}}{2}-11.0\text{ in.}\right)^2+984\text{ in.}^4+\left(14.7\text{ in.}^2\right)(11.0\text{ in.})^2 \\
&= 3,280\text{ in.}^4
\end{aligned}$$

Girder Mode Properties

The equivalent uniform loading for the girder is

$$\begin{aligned}
w_g &= L_j\left(\frac{w_j}{S}\right)+\text{ girder weight per unit length} \\
&= (35.0\text{ ft})\left(\frac{595\text{ plf}}{10.0\text{ ft}}\right)+50.0\text{ plf} \\
&= 2,130\text{ plf}
\end{aligned}$$

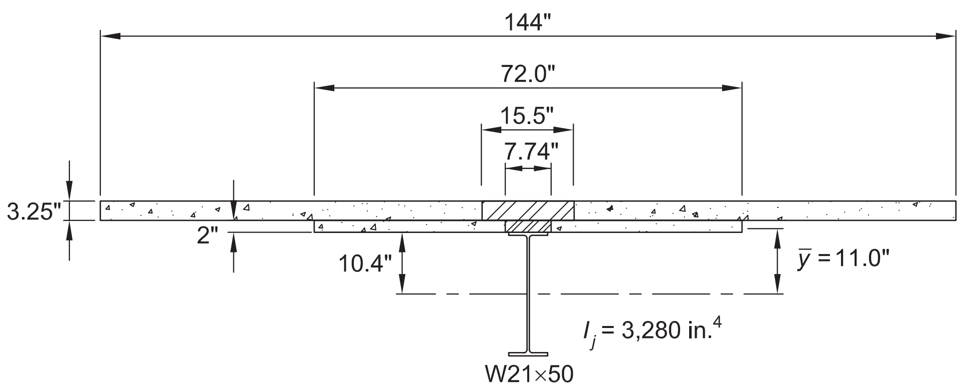


Fig. 4-7. Composite girder cross section for Example 4.1.

The corresponding deflection is

$$
\begin{aligned}
\Delta_g &= \frac{5w_g L_g^4}{384E_s I_g} \\
&= \frac{5(2{,}130 \text{ plf})(30.0 \text{ ft})^4 (1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{ psi})(3{,}280 \text{ in.}^4)} \\
&= 0.408 \text{ in.}
\end{aligned}
$$

From Equation 3-3, the girder mode fundamental frequency is

$$
f_g = 0.18\sqrt{\frac{g}{\Delta_g}} \tag{3-3}
$$

$$
\begin{aligned}
&= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.408 \text{ in.}}} \\
&= 5.54 \text{ Hz}
\end{aligned}
$$

With

$$
\begin{aligned}
D_j &= 183 \text{ in.}^4/\text{ft} \\
D_g &= \frac{I_g}{L_j} \\
&= \frac{3{,}280 \text{ in.}^4}{35.0 \text{ ft}} \\
&= 93.7 \text{ in.}^4/\text{ft} \\
C_g &= 1.8
\end{aligned}
$$

the effective girder panel width using Equation 4-4 is

$$
B_g = C_g\left(\frac{D_j}{D_g}\right)^{1/4} L_g \le \left(\frac{2}{3}\right) \textit{floor length} \tag{4-4}
$$

$$
\begin{aligned}
&= 1.8\left(\frac{183 \text{ in.}^4/\text{ft}}{93.7 \text{ in.}^4/\text{ft}}\right)^{1/4}(30.0 \text{ ft}) \\
&= 63.8 \text{ ft}
\end{aligned}
$$

Per Equation 4-4, the effective beam panel width must be less than or equal to two-thirds of the *floor length*. Because ⅔(105 ft) = 70.0 ft > 63.8 ft, the girder panel width is 63.8 ft. From Equation 4-2, the girder panel weight is

$$
\begin{aligned}
W_g &= \left(\frac{w_g}{L_j}\right) B_g L_g \\
&= \left(\frac{2{,}130 \text{ plf}}{35.0 \text{ ft}}\right)(63.8 \text{ ft})(30.0 \text{ ft}) \\
&= 116{,}000 \text{ lb}
\end{aligned}
$$

The girder panel weight was not increased by 50%, as was done in the joist panel weight calculation, because continuity effects generally are not realized when the girders frame directly into the column.

Combined Mode Properties

From Equation 3-4, the floor fundamental frequency is

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g}} \tag{3-4}$$

$$= 0.18\sqrt{\frac{386 \text{ in.}^2/\text{s}}{0.376 \text{ in.} + 0.408 \text{ in.}}}$$

$$= 3.99 \text{ Hz}$$

Because the girder span (30 ft) is less than the joist panel width (32.3 ft), the girder deflection, Δ_g, is reduced according to Equation 4-6. Because 30.0 ft/32.3 ft ≥ 0.5,

$$\Delta'_g = \frac{L_g}{B_j}\left(\Delta_g\right) \tag{4-6}$$

$$= \frac{30.0 \text{ ft}}{32.3 \text{ ft}}\left(0.408 \text{ in.}\right)$$

$$= 0.379 \text{ in.}$$

From Equation 4-5, the equivalent panel mode panel weight is

$$W = \frac{\Delta_j}{\Delta_j + \Delta'_g}W_j + \frac{\Delta'_g}{\Delta_j + \Delta'_g}W_g \qquad \text{(from Eq. 4-5)}$$

$$= \frac{0.376 \text{ in.}}{0.376 \text{ in.} + 0.379 \text{ in.}}(101{,}000 \text{ lb}) + \frac{0.379 \text{ in.}}{0.376 \text{ in.} + 0.379 \text{ in.}}(116{,}000 \text{ lb})$$

$$= 109{,}000 \text{ lb}$$

Evaluation

Using Equation 4-1 with P_o = 65 lb and β = 0.03:

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \tag{4-1}$$

$$= \frac{(65.0 \text{ lb})\left(e^{-0.35(3.99 \text{ Hz})}\right)}{0.03(109{,}000 \text{ lb})}$$

$$= 0.0049 \text{ equivalent to } 0.49\%g$$

The peak acceleration is less than the tolerance acceleration limit, a_p/g of 0.5%, as given in Table 4-1. The floor is therefore predicted to be satisfactory.

Example 4.2—Typical Interior Bay of an Office Building with Open-Web Joist/Hot-Rolled Girder Framing

Given:

The framing system shown in Figure 4-8 is to be evaluated for paper office occupancy. The office space will not have full-height partitions. The superimposed dead load, including mechanical equipment and ceiling, is assumed to be 4 psf. The live load is assumed to be 11 psf. The slab is 5 in. total depth, normal weight concrete (w_c = 145 psf, f'_c, = 3 ksi) on 1½-in.-deep deck. The joist bottom chords are not extended, and the joists are connected to the girder flange with joist seats. The damping ratio is β = 0.03. The *floor width* is 60 ft and the *floor length* is 90 ft. The assumed selfweight of the deck is 2 psf. The beams and girders are ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and Girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Girder
W24×55
A = 16.2 in.^{2}
I_x = 1,350 in.^{4}
d = 23.6 in.

The joist properties are as follows:

Joist
30K12

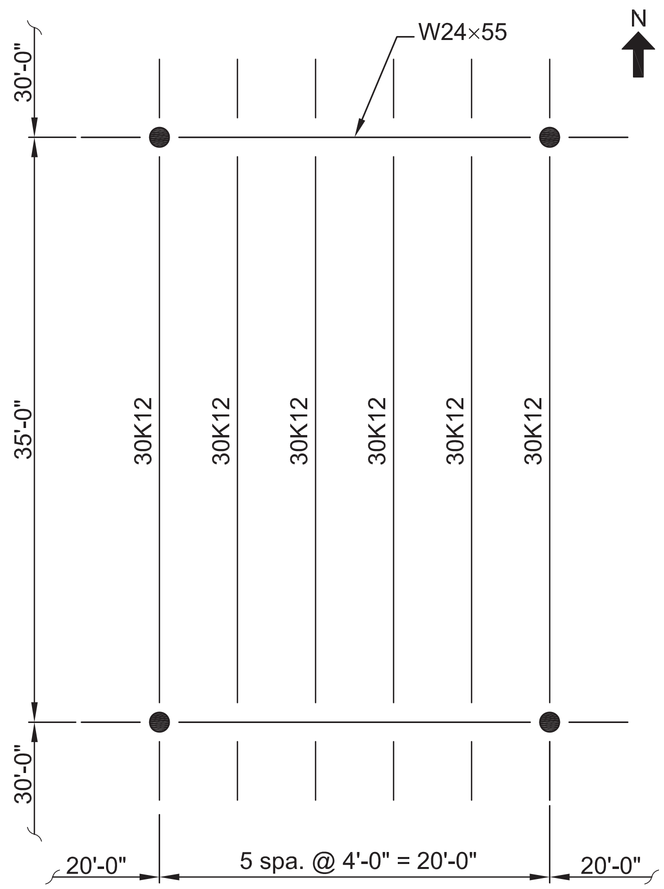


Fig. 4-8. Interior bay framing details for Example 4.2.

Selfweight = 17.6 plf
A = 1.91 in.^{2}
D = 30.0 in.
I_{chords} = 384 in.^{4}
y_c = 13.4 in.

From the recommended values given in Table 4-2, the estimated damping ratio is determined as follows:

$$\beta = 0.01(\text{structural system}) + 0.01\ (\text{ceiling and ductwork}) + 0.01\ (\text{paper office fit-out})$$
$$= 0.03$$

The properties of the deck are determined as follows:

$$\begin{aligned} E_c &= w^{1.5}\sqrt{f_c'} \\ &= (145 \text{ pcf})^{1.5}\sqrt{3 \text{ ksi}} \\ &= 3{,}020 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{slab} + \text{deck weight} &= \frac{\left(3.50 \text{ in.} + \dfrac{1.50 \text{ in.}}{2}\right)(145 \text{ pcf})}{12 \text{ in./ft}} + 2.00 \text{ psf} \\ &= 53.4 \text{ psf} \end{aligned}$$

Joist Transformed Effective Moment of Inertia

Considering only the concrete above the steel form deck and using a dynamic concrete modulus of elasticity of 1.35E_c, as discussed in Section 3.2, the modular ratio is

$$n = \frac{E_s}{1.35E_c} \tag{4-3c}$$
$$= \frac{29{,}000 \text{ ksi}}{1.35(3{,}020 \text{ ksi})}$$
$$= 7.11$$

As discussed in Section 3.2, the effective concrete slab width used for calculation of the transformed moment of inertia is determined as follows (refer to Figure 4-9):

$$\begin{aligned} \min[0.4L_i, S] &= \min[0.4(30.0 \text{ ft})(12 \text{ in./ft}), 48.0 \text{ in.}] \\ &= 48.0 \text{ in.} \end{aligned}$$

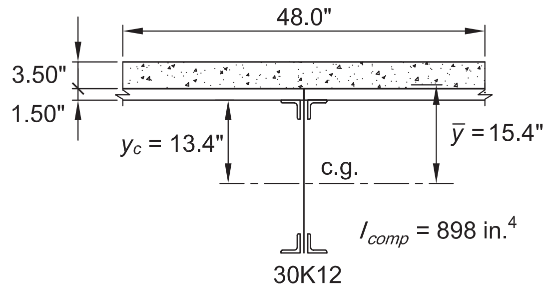


Fig. 4-9. Composite joist cross section for Example 4.2.

The transformed concrete slab width is

$$48.0\text{ in.}/7.11 = 6.75\text{ in.}$$

Using an effective concrete slab depth of 3.50 in., the transformed concrete slab area is

$$(3.50\text{ in.})(6.75\text{ in.}) = 23.6\text{ in.}^2$$

The composite transformed moment of inertia is computed as follows:

$$\begin{aligned}
\bar{y} &= \frac{\left(23.6\text{ in.}^2\right)\left[13.4\text{ in.} + 1\tfrac{1}{2}\text{ in.} + \left(\dfrac{3.50\text{ in.}}{2}\right)\right]}{23.6\text{ in.}^2 + 1.91\text{ in.}^2} \\
&= 15.4\text{ in. (above c.g. of joist)}
\end{aligned}$$

$$\begin{aligned}
I_{comp} &= \frac{(6.75\text{ in.})(3.50\text{ in.})^3}{12} + \left(23.6\text{ in.}^2\right)\left(13.4\text{ in.} + 1\tfrac{1}{2}\text{ in.} + \frac{3.50\text{ in.}}{2} - 15.4\text{ in.}\right)^2 + 384\text{ in.}^4 + \left(1.91\text{ in.}^2\right)(15.4\text{ in.})^2 \\
&= 898\text{ in.}^4
\end{aligned}$$

From Section 3.5, because $6 \le L_j/D = (30.0\text{ ft})(12\text{ in./ft})/(30.0\text{ in.}) = 12.0$, Equation 3-9a is applicable:

$$\begin{aligned}
C_r &= 0.90\left[1 - e^{-0.28\left(L_j/D\right)}\right]^{2.8} \qquad \text{(3-9a)} \\
&= 0.90\left[1 - e^{-0.28(12.0)}\right]^{2.8} \\
&= 0.815 < 0.9
\end{aligned}$$

Using Equation 3-8 and then 3-7, the effective joist transformed moment of inertia is

$$\begin{aligned}
\gamma &= \frac{1}{C_r} - 1 \qquad \text{(3-8)} \\
&= \frac{1}{0.815} - 1 \\
&= 0.227
\end{aligned}$$

$$\begin{aligned}
I_j &= \frac{1}{\dfrac{\gamma}{I_{chords}} + \dfrac{1}{I_{comp}}} \qquad \text{(3-7)} \\
&= \frac{1}{\left(\dfrac{0.227}{384\text{ in.}^4}\right) + \left(\dfrac{1}{898\text{ in.}^4}\right)} \\
&= 587\text{ in.}^4
\end{aligned}$$

For each joist, the uniformly distributed loading, including 11 psf live load and 4 psf dead load for mechanical/ceiling loads, is

$$\begin{aligned}
w_j &= (48.0\text{ in.})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)(11.0\text{ psf} + 53.4\text{ psf} + 4.00\text{ psf}) + 17.6\text{ plf} \\
&= 291\text{ plf}
\end{aligned}$$

The corresponding deflection is determined as follows:

$$\Delta_j = \frac{5w_j L_j^4}{384 E_s I_j}$$

$$= \frac{5(291\text{ plf})(30.0\text{ ft})^4(1{,}728\text{ in.}^3/\text{ft}^3)}{384(29\times10^6\text{ psi})(587\text{ in.}^4)}$$

$$= 0.312\text{ in.}$$

From Equation 3-3, the joist mode fundamental frequency is

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} \tag{3-3}$$

$$= 0.18\sqrt{\frac{386\text{ in./s}^2}{0.312\text{ in.}}}$$

$$= 6.33\text{ Hz}$$

Using an average concrete thickness of $d_e = 4.25$ in., the transformed moment of inertia per unit width in the slab span direction is

$$D_s = \frac{12 d_e{}^3}{12n} \tag{4-3a}$$

$$= \frac{(12\text{ in./ft})(4.25\text{ in.})^3}{12(7.11)}$$

$$= 10.8\text{ in.}^4/\text{ft}$$

The transformed moment of inertia per unit width in the joist span direction with joist spacing of 4 ft is

$$D_j = \frac{I_j}{S} \tag{4-3b}$$

$$= \frac{587\text{ in.}^4}{4.00\text{ ft}}$$

$$= 147\text{ in.}^4/\text{ft}$$

The effective beam panel width from Equation 4-3 with $C_j = 2.0$ is

$$B_j = C_j\left(\frac{D_s}{D_j}\right)^{1/4} L_j \le \left(\frac{2}{3}\right) \textit{floor width} \tag{4-3}$$

$$= 2.0\left(\frac{10.8\text{ in.}^4/\text{ft}}{147\text{ in.}^4/\text{ft}}\right)^{1/4}(30.0\text{ ft})$$

$$= 31.2\text{ ft}$$

Per Equation 4-3, the effective beam panel width must be less than or equal to two-thirds of the *floor width*. Because ⅔(60.0 ft) = 40.0 ft > 31.2 ft, the effective beam panel width is 31.2 ft.

The weight of the beam panel is calculated using Equation 4-2 without adjustment for continuity because the joist bottom chords are not extended.

$$W_j = \left(\frac{w_j}{S}\right) B_j L_j \qquad \text{(from Eq. 4-2)}$$

$$= \left(\frac{291 \text{ plf}}{4.00 \text{ ft}}\right)(31.2 \text{ ft})(30.0 \text{ ft})$$

$$= 68{,}100 \text{ lb}$$

Girder Transformed Effective Moment of Inertia

The effective concrete slab width is determined as follows:

$$\min\left[0.2\,L_g,\, 0.5L_{j,left}\right] + \min\left[0.2\,L_g,\, 0.5L_{j,right}\right] = \min\left[0.2(20.0 \text{ ft}),\, 0.5(30.0 \text{ ft})\right] + \min\left[0.2(20.0),\, 0.5(30.0 \text{ ft})\right]$$

$$= 4.00 \text{ ft} + 4.00 \text{ ft}$$

$$= 8.00 \text{ ft}$$

As shown in Figure 4-10, the transformed concrete slab width, using a dynamic concrete modulus of elasticity of $1.35E_c$, is

$$(8.00 \text{ ft})(12 \text{ in./ft})/7.11 = 13.5 \text{ in.}$$

Assuming that the deck has a symmetrical profile, the effective width of the slab in the deck is taken as 48 in. The transformed concrete width in the deck is

$$48.0 \text{ in.}/7.11 = 6.75 \text{ in.}$$

The transformed concrete slab area is

$$(3\tfrac{1}{2} \text{ in.})(13.5 \text{ in.}) = 47.3 \text{ in.}^2$$

The transformed concrete slab area in the deck is

$$(1\tfrac{1}{2} \text{ in.})(6.75 \text{ in.}) = 10.1 \text{ in.}^2$$

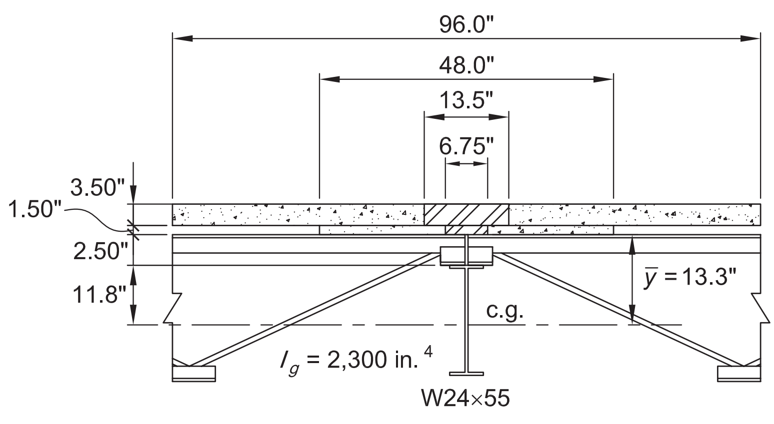


Fig. 4-10. Composite girder cross-section for Example 4.2.

The fully composite transformed moment of inertia is computed as follows:

$$\bar{y} = \frac{\left(47.3\text{ in.}^2\right)\left(\dfrac{23.6\text{ in.}}{2} + 2.50\text{ in.} + 1\tfrac{1}{2}\text{ in.} + \dfrac{3.50\text{ in.}}{2}\right) + \left(10.1\text{ in.}^2\right)\left(\dfrac{23.6\text{ in.}}{2} + 2.50\text{ in.} + \dfrac{1.50\text{ in.}}{2}\right)}{47.3\text{ in.}^2 + 10.1\text{ in.}^2 + 16.2\text{ in.}^2}$$

$$= 13.3\text{ in. (above c.g. of girder)}$$

$$I_{comp} = \frac{(13.5\text{ in.})(3.50\text{ in.})^3}{12} + \left(47.3\text{ in.}^2\right)\left(\frac{23.6\text{ in.}}{2} + 2.50\text{ in.} + 1\tfrac{1}{2}\text{ in.} + \frac{3.50\text{ in.}}{2} - 13.3\text{ in.}\right)^2 + \frac{(6.75\text{ in.})(1.50\text{ in.})^3}{12}$$

$$+ \left(10.1\text{ in.}^2\right)\left(\frac{23.6\text{ in.}}{2} + 2.50\text{ in.} + \frac{1.50\text{ in.}}{2} - 13.3\text{ in.}\right)^2 + 1{,}350\text{ in.}^4 + \left(16.2\text{ in.}^2\right)(13.3\text{ in.})^2$$

$$= 5{,}150\text{ in.}^4$$

To account for the reduced girder stiffness due to flexibility of the joist seats, I_g is reduced according to Equation 3-11:

$$I_g = I_x + \frac{I_{comp} - I_x}{4} \tag{3-11}$$

$$= 1{,}350\text{ in.}^4 + \left(\frac{5{,}150\text{ in.}^4 - 1{,}350\text{ in.}^4}{4}\right)$$

$$= 2{,}300\text{ in.}^4$$

Girder Mode Properties

For each girder, the equivalent uniform loading is

$$w_g = L_j\left(\frac{w_j}{S}\right) + \text{ girder weight per unit length}$$

$$= (30.0\text{ ft})\left(\frac{291\text{ plf}}{4.00\text{ ft}}\right) + 55.0\text{ plf}$$

$$= 2{,}240\text{ plf}$$

And the corresponding deflection is

$$\Delta_g = \frac{5w_g L_g^4}{384EI_g}$$

$$= \frac{5\left(2{,}240\text{ plf}\right)\left(20.0\text{ ft}\right)^4\left(1{,}728\text{ in.}^3/\text{ft}^3\right)}{384\left(29\times10^6\text{ psi}\right)\left(2{,}300\text{ in.}^4\right)}$$

$$= 0.121\text{ in.}$$

From Equation 3-3, the girder mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} \tag{3-3}$$

$$= 0.18\sqrt{\frac{386\text{ in./s}^2}{0.121\text{ in.}}}$$

$$= 10.2\text{ Hz}$$

With

$$
\begin{aligned}
D_j &= 147 \text{ in.}^4/\text{ft} \\
D_g &= \frac{I_g}{L_j} \\
&= \frac{2{,}300 \text{ in.}^4}{30.0 \text{ ft}} \\
&= 76.7 \text{ in.}^4/\text{ft} \\
C_g &= 1.6
\end{aligned}
$$

the effective girder panel width using Equation 4-4 is

$$
B_g = C_g \left(\frac{D_j}{D_g}\right)^{1/4} L_g \le \left(\frac{2}{3}\right) \textit{floor length} \tag{4-4}
$$

$$
\begin{aligned}
&= 1.6\left(\frac{147 \text{ in.}^4/\text{ft}}{76.7 \text{ in.}^4/\text{ft}}\right)^{1/4}(20.0 \text{ ft}) \\
&= 37.7 \text{ ft}
\end{aligned}
$$

From Equation 4-4, the effective width must be less than or equal to two-thirds of the *floor length*. Because ⅔(90.0 ft) = 60.0 ft > 37.7 ft, the girder *floor length* is taken as 37.7 ft. From Equation 4-2, the girder panel weight is

$$
W_g = \left(\frac{w_g}{L_j}\right) B_g L_g \qquad \text{(from Eq. 4-2)}
$$

$$
\begin{aligned}
&= \left(\frac{2{,}240 \text{ plf}}{30.0 \text{ ft}}\right)(37.7 \text{ ft})(20.0 \text{ ft}) \\
&= 56{,}300 \text{ lb}
\end{aligned}
$$

Combined Mode Properties

The combined mode frequency using Equation 3-4 is

$$
f_n = 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g}} \tag{3-4}
$$

$$
\begin{aligned}
&= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.312 \text{ in.} + 0.121 \text{ in.}}} \\
&= 5.37 \text{ Hz}
\end{aligned}
$$

The girder span (20 ft) is less than the effective joist panel width (B_j = 31.2 ft); therefore, the girder deflection, Δ_g, must be reduced. From Equation 4-6 and noting that 20.0 ft/31.2 ft = 0.641 ≥ 0.5

$$
\Delta'_g = \frac{L_g}{B_j}(\Delta_g) \tag{4-6}
$$

$$
\begin{aligned}
&= \left(\frac{20.0 \text{ ft}}{31.2 \text{ ft}}\right)(0.121 \text{ in.}) \\
&= 0.0776 \text{ in.}
\end{aligned}
$$

From Equation 4-5, the equivalent panel mode weight is

$$W = \frac{\Delta_j}{\Delta_j + \Delta'_g} W_j + \frac{\Delta'_g}{\Delta_j + \Delta'_g} W_g \qquad \text{(from Eq. 4-5)}$$

$$= \frac{0.312 \text{ in.}}{0.312 \text{ in.} + 0.0776 \text{ in.}}(68{,}100 \text{ lb}) + \frac{0.0776 \text{ in.}}{0.312 \text{ in.} + 0.0776 \text{ in.}}(56{,}300 \text{ lb})$$

$$= 65{,}700 \text{ lb}$$

Walking Evaluation

The peak acceleration is determined using Equation 4-1 with $P_o = 65$ lb and $\beta = 0.03$ as follows:

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \qquad (4\text{-}1)$$

$$= \frac{(65.0 \text{ lb})\left(e^{-0.35(5.37 \text{ Hz})}\right)}{0.03(65{,}700 \text{ lb})}$$

$$= 0.00503 \text{ equivalent to } 0.503\%g$$

The peak acceleration is equal to the tolerance acceleration limit, a_o/g of 0.5%, as given in Table 4-1. The floor is therefore predicted to be satisfactory.

Example 4.3—Mezzanine with Beam Edge Member

Given:

The mezzanine framing shown in Figure 4-11 is to be evaluated for walking vibrations. The floor system supports a paper office occupancy without full-height partitions. Note that framing details are the same as those for Example 4.1 except that the floor system is only one bay wide normal to the edge of the mezzanine floor. Also note that the edge member is a beam. The framing is

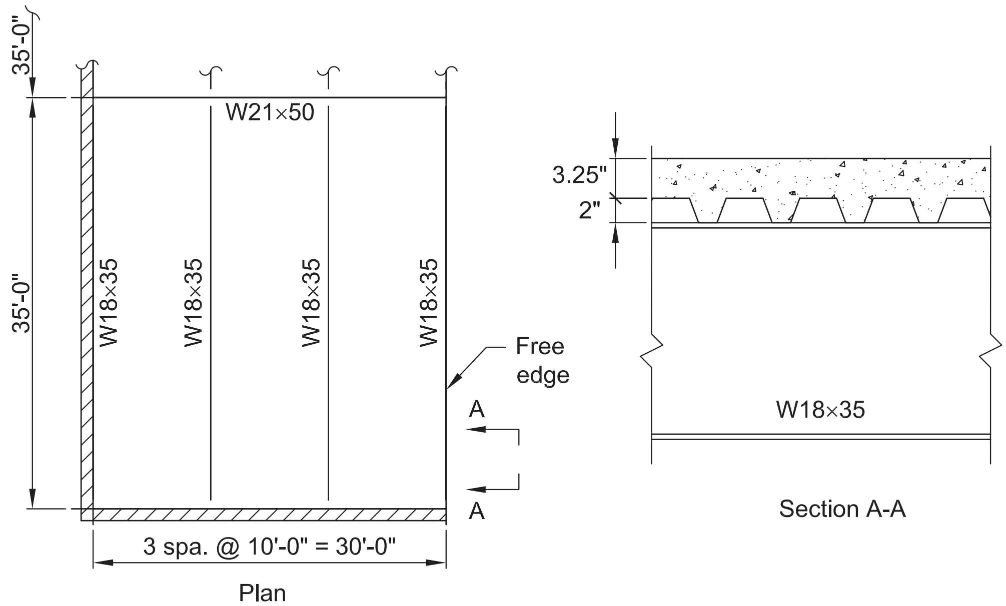


Fig. 4-11. Mezzanine with edge beam member framing details for Example 4.3.

analyzed using 11 psf live load and a superimposed dead load of 4 psf for the weight of mechanical equipment and ceiling. The damping ratio is $\beta = 0.03$. The slab is 5¼ in. total depth, lightweight concrete ($w_c = 110$ pcf and $f'_c = 4$ ksi) on 2-in.-deep deck. The *floor width* is 30 ft and the *floor length* is 105 ft.

Solution:

Beam Mode Properties

From Example 4.1:

$B_j = 32.3 \text{ ft}/2$ (because $C_j = 1.0$)
$\quad = 16.2$ ft for an unstiffened edge panel
$D_j = 183 \text{ in.}^4/\text{ft}$
$D_s = 8.25 \text{ in.}^4/\text{ft}$
$f_j = 5.77$ Hz
$w_j = 595$ plf
$\Delta_j = 0.376$ in.

From Equation 4-3, because the actual *floor width* is 30 ft and ⅔(30.0 ft) = 20.0 ft > 16.2 ft, the effective beam panel width is 16.2 ft.

The effective weight of the beam panel is calculated from Equation 4-2, adjusted by a factor of 1.5 to account for continuity in the beam direction:

$$\begin{aligned} W_j &= 1.5 w_j B_j L_j \\ &= 1.5(595 \text{ plf}/10.0 \text{ ft})(16.2 \text{ ft})(35.0 \text{ ft}) \\ &= 50{,}600 \text{ lb} \end{aligned}$$

Girder Mode Properties

From Example 4.1, with the *floor length* equal to 105 ft:

$B_g = 63.8$ ft
$W_g = 116{,}000$ lb
$f_g = 5.54$ Hz
$w_g = 2{,}130$ plf
$\Delta_g = 0.408$ in.

Combined Mode Properties

The combined mode frequency from Example 4.1 is 3.99 Hz.

In this case, the girder span (30 ft) is greater than the beam panel width (16.2 ft); thus, the girder deflection, Δ_g, is not reduced as was done in Example 4.1. From Equation 4-5,

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \tag{4-5}$$

$$\begin{aligned} &= \frac{0.376 \text{ in.}}{0.376 \text{ in.} + 0.408 \text{ in.}}(50{,}600 \text{ lb}) + \frac{0.408 \text{ in.}}{0.376 \text{ in.} + 0.408 \text{ in.}}(116{,}000 \text{ lb}) \\ &= 84{,}600 \text{ lb} \end{aligned}$$

Evaluation

Using Equation 4-1, with $P_o = 65$ lb and $\beta = 0.03$:

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \tag{4-1}$$

$$= \frac{(65.0 \text{ lb})\left(e^{-0.35(3.99 \text{ Hz})}\right)}{0.03(84{,}600 \text{ lb})}$$

$$= 0.00634 \text{ equivalent to } 0.634\%g$$

The peak acceleration is greater than the tolerance acceleration limit, a_o/g, of 0.5%, as given in Table 4-1. In this example, the edge member is a beam, and thus the beam panel width is half that of an interior bay. The result is that the combined panel does not have sufficient mass to satisfy the design criterion.

Example 4.4—Mezzanine with Girder Edge Member

Given:

The mezzanine framing shown in Figure 4-12 is to be evaluated for walking vibrations. All details are the same as in Example 4.3 except that the framing is rotated 90°. Note that the edge member is now a girder and that the basic framing is the same as used in Example 4.1. The mezzanine is assumed to be one bay wide normal to the edge girder. The framing is analyzed using 11 psf live load and a superimposed dead load of 4 psf for the weight of mechanical equipment and ceiling. The damping ratio is β = 0.03. The *floor width* is 60 ft, and the *floor length* is 35 ft.

Solution:

Beam Mode Properties

From Example 4.1:

B_j = 32.3 ft
D_j = 183 in.^{4}/ft
D_s = 8.25 in.^{4}/ft
f_j = 5.77 Hz
w_j = 595 plf
Δ_j = 0.376 in.

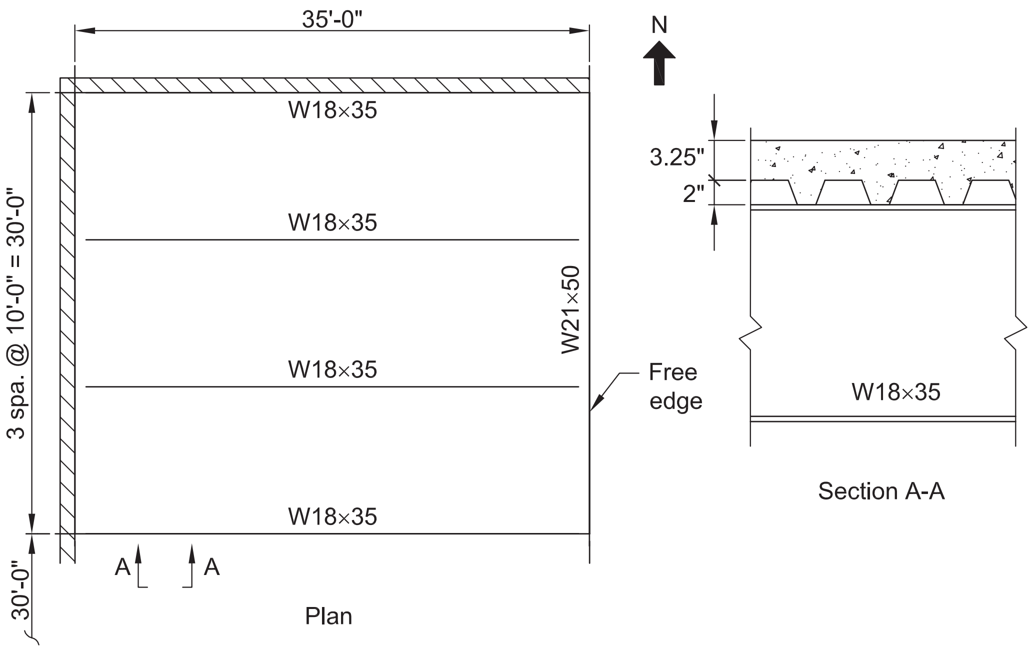


Fig. 4-12. Mezzanine with girder edge member framing details for Example 4.4.

From the framing plan, the actual *floor width* normal to the beams is at least 2(30.0 ft) = 60.0 ft. From Equation 4-3, because ⅔(60.0 ft) = 40.0 ft > 32.3 ft, the effective beam panel width is 32.3 ft.

The effective weight of the beam panel from Equation 4-2 is

$$\begin{aligned} W_j &= w_j B_j L_j \qquad (4\text{-}2) \\ &= \left(\frac{595 \text{ plf}}{10.0 \text{ ft}}\right)(32.3 \text{ ft})(35.0 \text{ ft}) \\ &= 67{,}300 \text{ lb} \end{aligned}$$

Girder Mode Properties

For the edge girder, the equivalent uniform loading is

$$\begin{aligned} w_g &= \frac{L_j}{2}\left(\frac{w_j}{S}\right) + \text{girder weight per unit length} \\ &= \left(\frac{35.0 \text{ ft}}{2}\right)\left(\frac{595 \text{ plf}}{10.0 \text{ ft}}\right) + 50.0 \text{ plf} \\ &= 1{,}090 \text{ plf} \end{aligned}$$

The transformed moment of inertia, assuming effective slab widths of 72 in. above the deck and 36 in. for concrete in the symmetrical profile deck, is I_g = 2,880 in.^{4} The corresponding deflection is

$$\begin{aligned} \Delta_g &= \frac{5 w_g L_g^4}{384 E_s I_g} \\ &= \frac{5(1{,}090 \text{ plf})(30.0 \text{ ft})^4 (1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{ psi})(2{,}880 \text{ in.}^4)} \\ &= 0.238 \text{ in.} \end{aligned}$$

As recommended in Section 4.1.3 for interior floor edges, the girder panel width is limited to two-thirds of the beam span and is determined as follows:

$$\begin{aligned} B_g &= \left(\frac{2}{3}\right) L_j \\ &= \left(\frac{2}{3}\right)(35.0 \text{ ft}) \\ &= 23.3 \text{ ft} \end{aligned}$$

From Equation 4-2, the girder panel weight is

$$\begin{aligned} W_g &= w_g B_g L_g \qquad (\text{from Eq. } 4\text{-}2) \\ &= \left(\frac{1{,}090 \text{ plf}}{17.5 \text{ ft}}\right)(23.3 \text{ ft})(30.0 \text{ ft}) \\ &= 43{,}500 \text{ lb} \end{aligned}$$

Combined Mode Properties

Using Equation 3-4, the combined mode frequency is

$$
\begin{aligned}
f_n &= 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g}} && \text{(3-4)}\\
&= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.376 \text{ in.} + 0.238 \text{ in.}}}\\
&= 4.51 \text{ Hz}
\end{aligned}
$$

In this case, the girder span (30 ft) is less than the joist panel width (32.3 ft), and the girder deflection, Δ_g, is therefore reduced according to Equation 4-6. Because 30.0 ft/32.3 ft ≥ 0.5 and ≤ 1.0,

$$
\begin{aligned}
\Delta'_g &= \frac{L_g}{B_j}\left(\Delta_g\right) && \text{(4-6)}\\
&= \left(\frac{30.0 \text{ ft}}{32.3 \text{ ft}}\right)(0.238 \text{ in.})\\
&= 0.221 \text{ in.}
\end{aligned}
$$

From Equation 4-5, the combined mode panel weight is

$$
\begin{aligned}
W &= \frac{\Delta_j}{\Delta_j + \Delta'_g}W_j + \frac{\Delta'_g}{\Delta_j + \Delta'_g}W_g && \text{(from Eq. 4-5)}\\
&= \frac{0.376 \text{ in.}}{0.376 \text{ in.} + 0.221 \text{ in.}}(67{,}300 \text{ lb}) + \frac{0.221 \text{ in.}}{0.376 \text{ in.} + 0.221 \text{ in.}}(43{,}500 \text{ lb})\\
&= 58{,}500 \text{ lb}
\end{aligned}
$$

Evaluation

Using Equation 4-1 with $P_o = 65$ lb and $\beta = 0.03$:

$$
\begin{aligned}
\frac{a_p}{g} &= \frac{P_o e^{-0.35 f_n}}{\beta W} && \text{(4-1)}\\
&= \frac{(65.0 \text{ lb})\left(e^{-0.35(4.51\text{Hz})}\right)}{0.03(58{,}500 \text{ lb})}\\
&= 0.00764 \text{ equivalent to } 0.764\%g
\end{aligned}
$$

The peak acceleration is greater than the acceleration limit, a_o/g, of 0.5%g, from Table 4-1. The floor is determined to be unsatisfactory in this example.

Because the mezzanine floor is only one bay wide normal to the edge girder, both the beams and the girder may need to be stiffened to satisfy the criterion.

Example 4.5—Pedestrian Bridge—Walking and Running

Given:

An outdoor pedestrian bridge of span 40 ft with pinned supports and the cross section shown in Figure 4-13 is to be evaluated for walking vertical and lateral vibration. Also, the vertical acceleration due to running on the bridge, using the method described in Section 2.4, is to be predicted. The bridge has a 6-in.-thick normal weight concrete slab ($f'_c = 4$ ksi, $w_c = 145$ pcf). Use $P_o = 92.0$ lb. The beams and girders are ASTM A992 material.

Solution:

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beams
W21×44
$A = 13.0$ in.^{2}
$I_x = 843$ in.^{4}
$I_y = 20.7$ in.^{4}
$d = 20.7$ in.

The properties of the deck are determined as follows:

$$\begin{aligned} E_c &= w^{1.5}\sqrt{f'_c} \\ &= (145 \text{ pcf})^{1.5}\sqrt{4 \text{ ksi}} \\ &= 3{,}490 \text{ ksi} \end{aligned}$$

$$\begin{aligned} n &= \frac{E_s}{1.35E_c} \qquad (4\text{-}3c) \\ &= \frac{29{,}000 \text{ ksi}}{1.35(3{,}490 \text{ ksi})} \\ &= 6.16 \end{aligned}$$

Slab weight = 72.5 psf

Because the pedestrian bridge is not supported by girders, only the beam panel mode needs to be investigated.

Beam Mode Properties

Because $0.4L_j = 0.4(40 \text{ ft}) = 16$ ft is greater than 5 ft, the full width of the slab is effective. Using a dynamic modulus of elasticity of $1.35E_c$, the transformed moment of inertia of both W21 beams combined is 5,830 in.^{4}

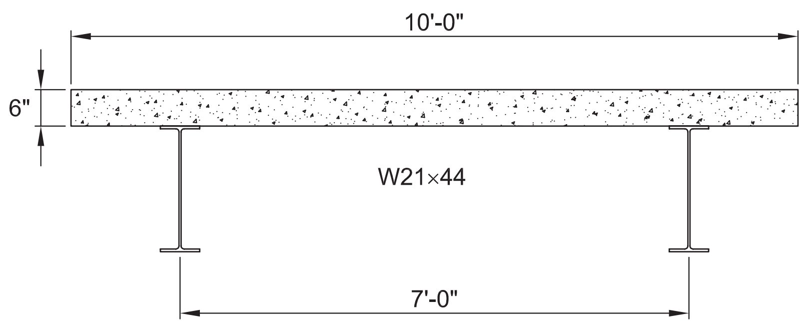


Fig. 4-13. Cross section for Example 4.5.

The bridge weight per linear ft is

$$w_j = (72.5 \text{ psf})(10.0 \text{ ft}) + 2(44.0 \text{ plf})$$
$$= 813 \text{ plf}$$

The corresponding deflection is determined as follows:

$$\Delta_j = \frac{5 w_j L_j^4}{384 E_s I_j}$$
$$= \frac{5(813 \text{ plf})(40.0 \text{ ft})^4 (1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29 \times 10^6 \text{ psi})(5{,}830 \text{ in.}^4)}$$
$$= 0.277 \text{ in.}$$

The beam mode fundamental frequency from Equation 3-3 is

$$f_j = 0.18\sqrt{\frac{g}{\Delta_j}} \tag{3-3}$$
$$= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.277 \text{ in.}}}$$
$$= 6.72 \text{ Hz}$$

The effective beam panel width, B_j, is 10 ft because the entire footbridge will vibrate as a simple beam. The weight of the beam panel is then

$$W_j = w_j L_j$$
$$= (813 \text{ plf})(40.0 \text{ ft})$$
$$= 32{,}500 \text{ lb}$$

Evaluation for Vertical Vibration Due to Walking

As discussed in Section 4.2, $\beta = 0.01$ for outdoor footbridges.

From Equation 4-1, with $f_n = f_j = 6.72$ Hz and $P_o = 92$ lb

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \tag{4-1}$$
$$= \frac{(92.0 \text{ lb})\left(e^{-0.35(6.72 \text{ Hz})}\right)}{0.01(32{,}500 \text{ lb})}$$
$$= 0.0269 \text{ equivalent to } 2.69\%g$$

The peak acceleration is less than the acceleration limit 5%g for outdoor footbridges per Table 4-4. The footbridge is therefore satisfactory for a single walker. The number of random walkers near the center of the bridge to cause an acceleration of 5%g is:

$$n = \left(\frac{5.0\%g}{2.7\%g}\right)^2$$
$$= 3.43$$

Evaluation for Vertical Vibration due to Running

From Equation 2-16, with $f_n = f_j = 6.72$ Hz and $Q = 168$ lb:

$$\frac{a_p}{g} = \frac{0.79Qe^{-0.173 f_n}}{\beta W} \leq \frac{a_o}{g} \tag{2-16}$$

$$= \frac{0.79(168 \text{ lb})\left(e^{-0.173(6.72 \text{ Hz})}\right)}{0.01(32{,}500 \text{ lb})}$$

$$= 0.128 \text{ equivalent to } 12.8\%g$$

This acceleration ratio exceeds the limit for outdoor pedestrian bridges, as shown in Figure 2-1, and therefore is judged unacceptable for running activities with stationary people on the bridge.

Evaluation for Lateral Vibration

Conservatively considering only beam stiffness, the moment of inertia for lateral vibration with beam spacing of 84 in. is

$$I_y \approx 2\left(20.7 \text{ in.}^4\right) + 2\left(13.0 \text{ in.}^2\right)\left(42.0 \text{ in.}\right)^2$$
$$= 45{,}900 \text{ in.}^4$$

The natural frequency of lateral vibration from Equation 3-1 is

$$f_n = \frac{\pi}{2}\left(\frac{gE_sI_t}{wL^4}\right)^{1/2} \tag{3-1}$$

$$= \frac{\pi}{2}\left[\frac{\left(386 \text{ in./s}^2\right)\left(29\times10^6 \text{ psi}\right)\left(45{,}900 \text{ in.}^4\right)}{\left(813 \text{ plf}\right)\left(40.0 \text{ ft}\right)^4\left(1{,}728 \text{ in.}^3/\text{ft}^3\right)}\right]^{1/2}$$

$$= 18.8 \text{ Hz}$$

The natural frequency of lateral vibration exceeds the minimum recommended lateral frequency for walking, 1.3 Hz, by a wide margin, so the bridge satisfies the lateral vibration criterion. Assuming a minimum lateral frequency for running of 2.5 Hz, which is greater than one-half a running speed of 4 Hz, the bridge is satisfactory for this loading case.

Example 4.6—Linear Stair—Individual and Group Loadings

Given:

The slender monumental stair shown in Figure 4-14 is to be evaluated for walking vibration. The parallel stringers are 60 in. center-to-center. Each stringer is an HSS20×12×⅜. The stair has a drywall soffit and the guardrails are constructed of steel studs and drywall.

The design assumptions used for this example are as follows:

W_s = weight of stair including guardrails, treads, stringers and soffit = 15,500 lb
β = damping ratio = 0.03
L_s = diagonal distance between supports = 36.9 ft

Solution:

From AISC *Manual* Table 1-11, the geometric properties are as follows:

Stringer
HSS20×12×⅜
$A = 21.5$ in.^{2}
$I_x = 1{,}200$ in.^{4}
$I_y = 547$ in.^{4}

Vertical Natural Frequency

$$I_x = 2\left(1{,}200 \text{ in.}^4\right)$$
$$= 2{,}400 \text{ in.}^4$$

$$f_n = \frac{\pi}{2}\left(\frac{gE_sI_x}{W_sL_s^3}\right)^{½} \qquad \text{(Eq. 4-7)}$$
$$= \frac{\pi}{2}\left[\frac{\left(386 \text{ in./s}^2\right)\left(29\times10^6 \text{ psi}\right)\left(2{,}400 \text{ in.}^4\right)}{(15{,}500 \text{ lb})(36.9 \text{ ft})^3\left(1{,}728 \text{ in.}^3/\text{ft}^3\right)}\right]^{½}$$
$$= 7.02 \text{ Hz}$$

As discussed in Section 4.3, because the stair vertical natural frequency is greater than 5.0 Hz, the design is satisfactory for this check.

Horizontal Natural Frequency

Conservatively assuming that stringer minor-axis bending provides all horizontal stiffness:

$$I_y = 2\left(547 \text{ in.}^4\right)$$
$$= 1{,}090 \text{ in.}^4$$

$$f_n = \frac{\pi}{2}\left(\frac{gE_sI_y}{W_sL_s^3}\right)^{½} \qquad \text{(Eq. 4-7)}$$
$$= \frac{\pi}{2}\left[\frac{\left(386 \text{ in./s}^2\right)\left(29\times10^6 \text{ psi}\right)\left(1{,}090 \text{ in.}^4\right)}{(15{,}500 \text{ lb})(36.9 \text{ ft})^3\left(1{,}728 \text{ in.}^3/\text{ft}^3\right)}\right]^{½}$$
$$= 4.73 \text{ Hz}$$

As discussed in Section 4.3, because the stair horizontal natural frequency is greater than 2.5 Hz, the design is satisfactory for this check.

Evaluation Criterion for Individual Descending at Normal Speeds

The walker and the affected occupant locations are shown in Figure 4-15.

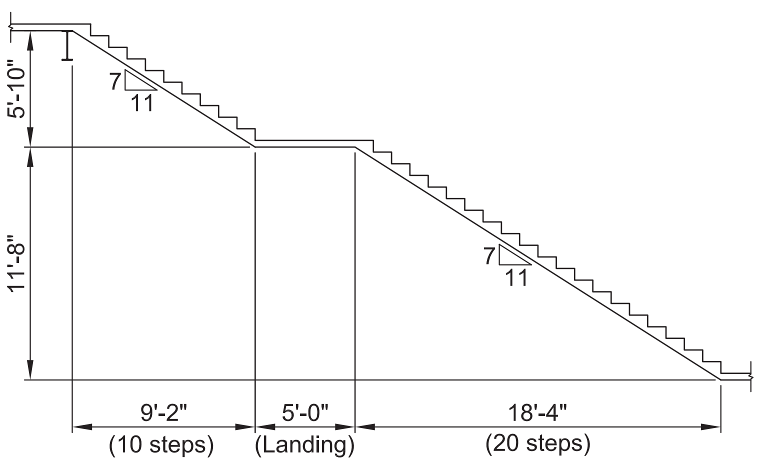


Fig. 4-14. Linear stair geometry for Example 4.6.

The mode shape amplitudes, ϕ_W and ϕ_R, using Equations 4-9 and 4-10 with $x_W = 17.3$ ft and $x_R = 15.1$ ft are

$$\phi_W = \sin\left(\frac{\pi x_W}{L_s}\right) \tag{4-9}$$

$$= \sin\left[\frac{\pi(17.3\text{ ft})}{36.9\text{ ft}}\right]$$

$$= 0.995$$

$$\phi_R = \sin\left(\frac{\pi x_R}{L_s}\right) \tag{4-10}$$

$$= \sin\left[\frac{\pi(15.1\text{ ft})}{36.9\text{ ft}}\right]$$

$$= 0.960$$

The predicted acceleration ratio from Equation 4-8 with $R = 0.7$ and $\gamma = 0.29$ from Table 4-5 is

$$\frac{a_p}{g} = 0.62e^{-\gamma f_n}\frac{RQ\cos^2\theta}{\beta W_s}\phi_W\phi_R\left(1-e^{-100\beta}\right) \le \frac{a_o}{g}$$

$$\text{with } \theta = \tan^{-1}\left(\frac{17.5\text{ ft}}{32.5\text{ ft}}\right) \tag{4-8}$$

$$= 28.3°$$

$$\frac{a_p}{g} = 0.62e^{-0.29(7.02\text{ Hz})}\frac{0.7(168\text{ lb})\left(\cos^2 28.3°\right)}{0.03(15{,}500\text{ lb})}(0.995)(0.960)\left[1-e^{-100(0.03)}\right]$$

$$= 0.0144 \text{ or } 1.44\%g$$

The predicted peak acceleration does not exceed the Table 4-5 tolerance limit, 1.7%g; thus, individuals descending the stair at normal speeds are not expected to cause objectionable vibrations from people standing on the stair.

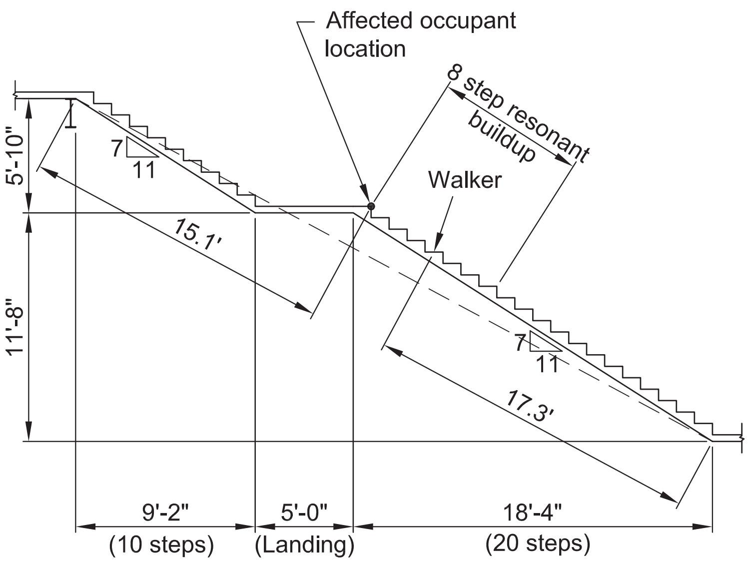


Fig. 4-15. Walker and affected occupant locations.

Evaluation Criterion for Individual Descending Rapidly

The predicted acceleration ratio from Equation 4-8 with $R = 0.5$ and $\gamma = 0.19$ from Table 4-5 is

$$\frac{a_p}{g} = 0.62e^{-\gamma f_n} \frac{RQ\cos^2\theta}{\beta W_s} \phi_W \phi_R \left(1 - e^{-100\beta}\right) \le \frac{a_o}{g} \tag{4-8}$$

$$= 0.62e^{-0.19(7.02\text{ Hz})} \frac{0.5(168\text{ lb})(\cos^2 28.3°)}{0.03(15{,}500\text{ lb})}(0.995)(0.960)\left[1 - e^{-100(0.03)}\right]$$

$$= 0.0208 \text{ or } 2.08\%g$$

The predicted peak acceleration does not exceed the Table 4-5 tolerance limit, $3\%g$; thus, individuals rapidly descending the stair are not expected to cause objectionable vibrations from people standing on the stair.

Evaluation Criterion for Rapidly Descending Group

The predicted peak acceleration due to a rapidly descending group is triple the acceleration due to a rapidly descending individual:

$$a_p = 3(2.08\%g)$$
$$= 6.23\%g$$

This acceleration exceeds the Table 4-5 tolerance limit of $4.5\%g$. Consequently, this loading is expected to cause strongly perceptible vibrations. The stair is unsatisfactory if this load case is to be considered.

Chapter 5
Design for Rhythmic Excitation

Rhythmic activities have the potential for causing significant vibration problems in buildings. This chapter provides design guidance for floors supporting dancing, lively crowd movement, and aerobics. Aerobics is probably the most severe occupant-caused serviceability loading in buildings and is known to have resulted in objectionable vibrations in a number of floors above an exercise area in tall buildings. The primary concern is the tolerance of occupants to floor motion near rhythmic activities. However, from collapses of dance floors and grandstands due to synchronized crowd movement, design rules that consider both ultimate and serviceability limit state criteria are now available; this is particularly true in the United Kingdom where, following a number of collapses (most notably, a temporary grandstand at a rock concert in 1997), the loading standard BS 6399-1:1996 was revised to include a section on synchronized dynamic crowd loads (BSI, 1996).

This chapter is limited to recommendations for evaluation of rhythmic excitation of floors and balconies. For grandstands in stadia, the reader is referred to *Dynamic Performance Requirements for Permanent Grandstands Subject to Crowd Motion* (IStructE, 2008).

5.1 RECOMMENDED EVALUATION CRITERIA

The recommended evaluation criteria states that the floor system is satisfactory if the peak acceleration response ratio, a_p/g, due to rhythmic activities predicted using the inequality shown in Equation 5-1 does not exceed the tolerance limits in Table 5-1. The predicted peak acceleration ratio, a_p/g, from a rhythmic activity is a combination (1.5 Power Rule) of the floor responses to each harmonic of the dynamic force (Allen, 1990b):

$$\frac{a_p}{g} = \frac{\left(\sum a_{p,i}^{1.5}\right)^{1/1.5}}{g} \le \frac{a_o}{g} \tag{5-1}$$

where a_o/g is the tolerance limit expressed as an acceleration ratio (Table 5-1) and the peak acceleration due to the ith harmonic is computed using the following equation (Allen et al., 1987).

$$\frac{a_{p,i}}{g} = \frac{1.3\alpha_i\, w_p/w_t}{\sqrt{\left[\left(\dfrac{f_n}{if_{step}}\right)^2 - 1\right]^2 + \left(\dfrac{2\beta f_n}{if_{step}}\right)^2}} \tag{5-2}$$

where

f_n = fundamental natural frequency, Hz

f_{step} = step frequency, Hz

i = harmonic number, 1, 2, 3

w_p = unit weight of rhythmic activity participants distributed over the entire bay, psf

w_t = distributed weight supported, including dead load, superimposed dead load, occupants, and participants distributed over the entire bay, psf

α_i = dynamic coefficient for the ith harmonic of the rhythmic activity (Table 5-2)

β = damping ratio, usually taken as 0.06 for rhythmic crowd loading

Occupants, as noted in the definition of w_t, are live load in the area not subject to the activity that must be considered, for example, live load in the dining area around a dance floor.

Equation 5-2 predicts the acceleration at midbay of framing supported by columns or walls and is not applicable to cantilevers. If the potentially affected occupants are away from midbay, the response can be multiplied by the unity-normalized mode shape values at the location of the occupants. The mode shapes can be determined from finite element analysis as shown in Chapter 7. If column deformation is not considered, i.e., $\Delta_c = 0$, the mode shape can be predicted using Equation 6-2.

The recommended tolerance acceleration limits due to rhythmic activities depend on the "affected occupancy," i.e., the occupancy in the rhythmic activity bay or adjacent bays. The recommended tolerance limits in Table 5-1 are from the 2010 National Building Code of Canada (NRCC, 2010a).

In the first edition of this Design Guide, design guidance was given in the form of a table for initial evaluation, a criterion based on acceptable frequency, and a criterion based on acceleration tolerance. Acceleration tolerance has been found to provide the best prediction of occupant response, and it is therefore the only evaluation method in this version of the Design Guide.

If the criteria recommended in this chapter are satisfied, fatigue will generally not be an issue for common structural floor systems.

5.2 ESTIMATION OF PARAMETERS

The most important structural parameter that must be considered in preventing building vibration problems due to rhythmic activities is the fundamental natural frequency of

Table 5-1. Recommended Tolerance Acceleration Limits for Rhythmic Activities in Buildings	
Affected Occupancy	**Tolerance Acceleration Limit, a_o, %g**
Office or residential	0.5
Dining	1.5–2.5
Weightlifting	1.5–2.5
Rhythmic activity only	4–7

Table 5-2. Dynamic Loading Parameters for Rhythmic Events			
Activity	**Harmonic Frequency, if_{step}, Hz**	**Distributed Weight of Participants, w_p, psf**	**Dynamic Coefficient, α_i**
Dancing:		12.5 (25 ft^2 per couple)	
First harmonic	1.5–2.7		0.50
Second harmonic	3.0–5.4		0.05
Lively concert (fixed seating):		31.0 (5 ft^2 per person)	
First harmonic	1.5–2.7		0.25
Second harmonic	3.0–5.4		0.05
Aerobics:		4.20 (35 ft^2 per person)	
First harmonic	2.0–2.75		1.5
Second harmonic	4.0–5.50		0.6
Third harmonic	6.0–8.25		0.1

vertical vibration of the structure, f_n. Also important is the loading function of the activity (Table 5-2) and the transmission of vibration to sensitive occupancies of the building. Of lesser importance are the supported weight, w_t, and the damping ratio, β.

Fundamental Natural Frequency, f_n

The floor fundamental natural frequency is much more important in relation to rhythmic excitation than it is for walking excitation, and therefore, more care is required for its estimation. For determining fundamental natural frequency, it is important to keep in mind that the participating structure extends all the way down to the foundations and even into the ground. Equation 3-5 can be used to estimate the natural frequency of the structure, including the beams or joists, girders and columns, and is repeated here for convenience.

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g + \Delta_c}} \tag{3-5}$$

where

Δ_c = axial shortening of the supporting column or wall due to the weight supported, in.

Δ_g, Δ_j = beam or joist and girder deflections determined as described in Chapter 3, in.

The contribution of column deflection, Δ_c, is generally small compared to joist and girder deflections for buildings with few (one to five) stories but becomes significant for buildings with many (more than six) stories because of the increased length of the column "spring." For high-rise buildings (more than 15 stories), the natural frequency due to the column springs alone may be in resonance with the second harmonic of the jumping frequency (Allen, 1990a; Lee et al., 2013).

A more accurate estimate of natural frequency may be obtained by finite element modeling of the structural system using the methods described in Chapter 7.

Acceleration Limit, a_o/g

The acceleration tolerance limits in Table 5-1 are for the affected occupancies on the activity floor. The fundamental mode shape can be used to estimate accelerations in other sensitive locations in the building and those estimated accelerations are then compared to the tolerance limits in Table 5-1.

Rhythmic Loading Parameters, w_p, α_i, and f_{step}

For the area used by the rhythmic activity, the distributed weight of participants, w_p, can be estimated from Table 5-2. In cases where participants occupy only part of the span, the value of w_p is reduced on the basis of equivalent effect (moment or deflection) for a fully loaded span (see also Hanagan, 2002). Values of α_i and if_{step} are recommended in Table 5-2.

Damping Ratio, β

Because participants contribute to the damping, a value of approximately 0.06 may be used. It is noted that changes in the relatively small damping ratios estimated for floors have very little effect on the predicted acceleration ratio from Equation 5-2 unless the harmonic forcing frequency, if_{step}, is equal to the floor natural frequency, f_n.

5.3 APPLICATION OF THE EVALUATION CRITERIA AND EXAMPLES

The designer should initially determine whether rhythmic activities are contemplated in the building, and if so, where. At an early stage in the design process, it is possible to locate both rhythmic activities and sensitive occupancies so as to minimize potential vibration problems and the costs required to avoid them. It is also a good idea at this stage to consider alternative structural solutions to prevent vibration problems. Such structural solutions may include design of the structure to control the accelerations in the building; special approaches, such as isolation of the activity floor from the rest of the building; or the use of mitigating devices, such as tuned mass dampers or floating floors.

Example 5.1—Long Span Joist-Supported Floor Used for Dining

Given:

The floor layout and framing shown in Figure 5-1 is a ballroom used for dining adjacent to the dancing area shown. The floor system consists of composite joists spanning 45 ft supported on concrete block walls. The width of the ballroom is 72 ft. The total weight of the floor is estimated to be 75 psf, including 15 psf for furnishings, and people dancing and dining. The effective composite moment of inertia of each joist is 2,600 in.^{4}

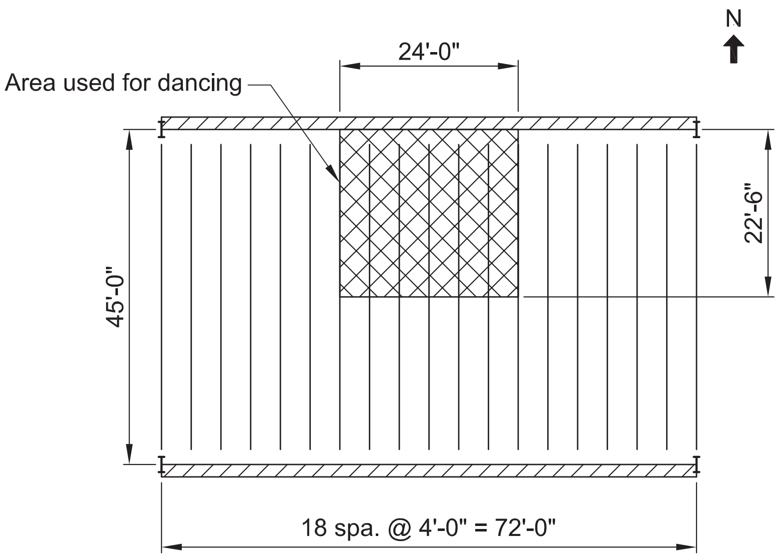


Fig. 5-1. Framing and layout of dance floor for Example 5.1.

Solution:

Natural Frequency

The deflection of a composite joist due to the supported 75 psf loading is

$$\begin{aligned}\Delta_j &= \frac{5w_jL_j^4}{384E_sI_j}\\ &= \frac{5(75.0\text{ psf})(4.00\text{ ft})(45.0\text{ ft})^4\left(1{,}728\text{ in.}^3/\text{ft}^3\right)}{384\left(29\times10^6\text{ psi}\right)\left(2{,}600\text{ in.}^4\right)}\\ &= 0.367\text{ in.}\end{aligned}$$

Because there are no girders, $\Delta_g = 0$, and because the axial deformation of the wall can be neglected, $\Delta_c = 0$. Thus, the fundamental natural frequency of the floor, from Equation 3-5, is approximately

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j+\Delta_g+\Delta_c}} \tag{3-5}$$

$$\begin{aligned}&= 0.18\sqrt{\frac{386\text{ in./s}^2}{0.367\text{ in.}+0\text{ in.}+0\text{ in.}}}\\ &= 5.84\text{ Hz}\end{aligned}$$

Predicted Peak Acceleration

The dance area is less than the total floor area; therefore, the weight of the participants from Table 5-2, 12.5 psf, can be reduced in proportion to the total floor area:

$$w_p = (12.5\text{ psf})\left[\frac{(22.5\text{ ft})(24.0\text{ ft})}{(45.0\text{ ft})(72.0\text{ ft})}\right] = 2.08\text{ psf}$$

The peak accelerations due to the 1st and 2nd harmonics of the dynamic force, as required in Table 5-2, are determined from Equation 5-2 with $w_t = 75.0$ psf, $w_p = 2.08$ psf, $\beta = 0.06$, $i = 1, 2$ and with $\alpha_1 = 0.50$ and $\alpha_2 = 0.05$ from Table 5-2. The predicted peak accelerations for step frequencies from 1.5 to 2.7 Hz using Equations 5-1 and 5-2 are shown in Table 5-3 and plotted in Figure 5-2.

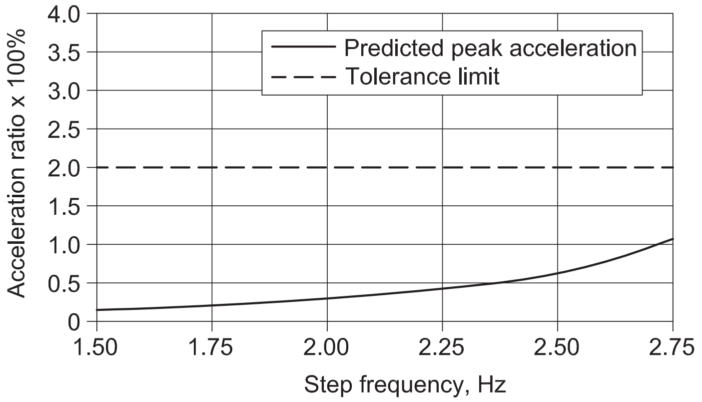


Fig. 5-2. Peak acceleration versus step frequency results for Example 5.1.

Table 5-3. Example 5.1 Peak Accelerations				
1st Harmonic		2nd Harmonic		Combined Peak
$1f_{step}$, Hz	a_1/g, %g	$2f_{step}$, Hz	a_2/g, %g	a_p, %g
1.50	0.13	3.00	0.06	0.16
1.60	0.15	3.20	0.08	0.18
1.70	0.17	3.40	0.09	0.21
1.80	0.19	3.60	0.11	0.24
1.90	0.21	3.80	0.13	0.28
2.00	0.24	4.00	0.16	0.32
2.10	0.27	4.20	0.19	0.37
2.20	0.30	4.40	0.23	0.42
2.30	0.33	4.60	0.29	0.49
2.40	0.37	4.80	0.36	0.58
2.50	0.40	5.00	0.46	0.69
2.60	0.44	5.20	0.61	0.85
2.70	0.49	5.40	0.85	1.08

Evaluation

From Table 5-1, a tolerance acceleration limit of 2%g is selected; i.e., $a_o/g = 0.02$. The maximum predicted peak acceleration occurs at a step frequency of 2.70 Hz and is 1.08%g, which is less than this selected tolerance acceleration limit. Therefore, the framing is satisfactory for dining and dancing for the size of the dance floor shown in Figure 5-1.

Example 5.2—Second Floor of General Purpose Building Used for Aerobics

Given:

Aerobics is to be considered in a bay of an occupied second floor of a six-story office building. The structural plan shown in Figure 5-3 satisfies all strength requirements. The floor construction consists of a concrete slab on hot-rolled beams, supported on hot-rolled girders and steel columns. The floor slab is 5½ in. total depth, normal weight concrete ($f'_c = 4.0$ ksi) on 2-in.-deep deck. The weight of the slab and deck is 56.4 psf. There are ceiling and ductwork below with an estimated weight of 4 psf. From Table 5-2, the weight of the aerobic participants is estimated to be 4.2 psf. The effective composite moments of inertia of the beams and girder are 1,920 in.^{4} and 4,740 in.^{4}, respectively. (See Example 4.1 for calculation procedures.) The spandrel girder is supported by the exterior cladding for vibration analysis purposes.

Solution:

The uniform load supported by a beam is

$$\begin{aligned} w_b &= (4.00 \text{ psf} + 56.4 \text{ psf} + 4.20 \text{ psf})(7.50 \text{ ft}) + 35.0 \text{ plf} \\ &= 520 \text{ plf} \end{aligned}$$

The equivalent uniform load supported by the girder assuming 11 psf live load on the adjacent bay is

$$\begin{aligned} w_g &= \left(\frac{520 \text{ plf}}{7.50 \text{ ft}}\right)\left(\frac{36.0 \text{ ft}}{2}\right) + (4.00 \text{ psf} + 56.4 \text{ psf} + 11.0 \text{ psf})\left(\frac{15.0 \text{ ft}}{2}\right) + 55.0 \text{ plf} \\ &= 1{,}840 \text{ plf} \end{aligned}$$

Natural Frequency

The natural frequency of the system is estimated by use of Equation 3-5. The required deflections due to the weight supported by each element (beams, girders and columns) are determined in the following.

The deflection of the beams due to the supported weight is

$$\begin{aligned}\Delta_b &= \frac{5 w_b L_b^4}{384 E_s I_b} \\ &= \frac{5(520 \text{ plf})(36.0 \text{ ft})^4 (1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{ psi})(1{,}920 \text{ in.}^4)} \\ &= 0.353 \text{ in.}\end{aligned}$$

The deflection of the girders due to the supported weight is

$$\begin{aligned}\Delta_g &= \frac{5 w_g L_g^4}{384 E_s I_g} \\ &= \frac{5(1{,}840 \text{ plf})(30.0 \text{ ft})^4 (1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{ psi})(4{,}740 \text{ in.}^4)} \\ &= 0.244 \text{ in.}\end{aligned}$$

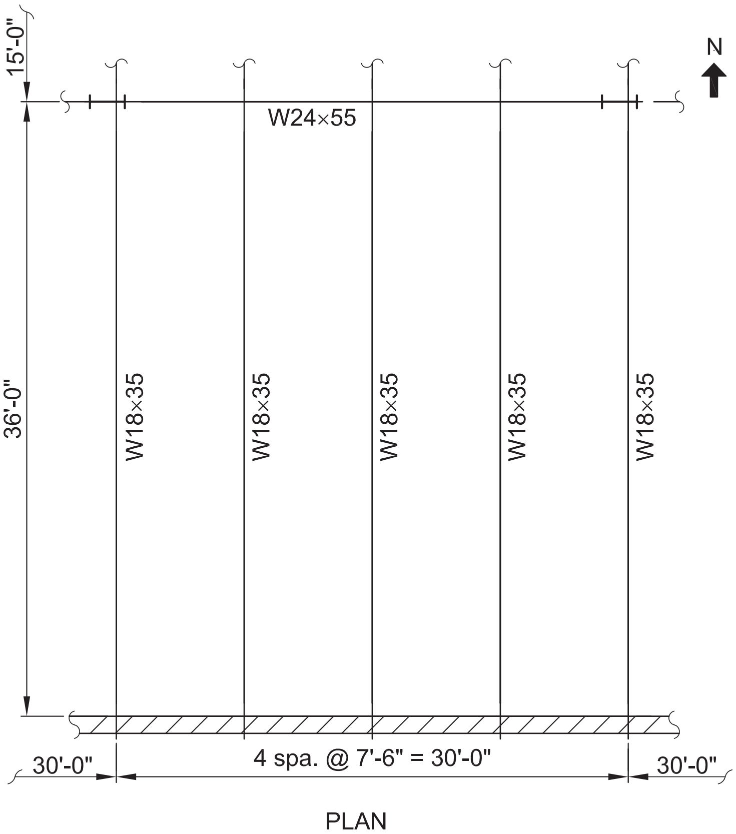


Fig 5-3. Aerobics floor structural layout for Example 5.2.

Table 5-4. Example 5.2 Peak Accelerations						
1st Harmonic		2nd Harmonic		3rd Harmonic		Peak
$1f_{step}$, Hz	a_1/g, %g	$2f_{step}$, Hz	a_2/g, %g	$3f_{step}$, Hz	a_3/g, %g	a_p, %g
2.0	2.96	4.0	17.4	6.0	1.70	18.5
2.1	3.35	4.2	26.8	6.3	1.53	27.9
2.2	3.79	4.4	38.1	6.6	1.41	39.1
2.23	**3.93**	**4.44**	**39.0**	**6.69**	**1.38**	**40.1**
2.3	4.26	4.6	35.0	6.9	1.32	36.2
2.4	4.80	4.8	25.9	7.2	1.25	27.4
2.5	5.39	5.0	19.9	7.5	1.19	21.9
2.6	6.05	5.2	16.2	7.8	1.15	18.8
2.7	6.80	5.4	13.9	8.1	1.11	17.1
2.75	7.21	5.5	13.0	8.25	1.09	16.6

The axial shortening of the columns is calculated from the axial stress due to the weight supported. Assuming an axial stress, f_a, of 6 ksi and a first floor column length of 16 ft:

$$\begin{aligned}\Delta_c &= \frac{f_a L_c}{E} \\ &= \frac{(6.00\text{ ksi})(16.0\text{ ft})(12\text{ in./ft})}{29{,}000\text{ ksi}} \\ &= 0.0397\text{ in.}\end{aligned}$$

The natural frequency from Equation 3-5 is

$$f_n = 0.18\sqrt{\frac{g}{\Delta_b + \Delta_g + \Delta_c}} \tag{3-5}$$

$$\begin{aligned}&= 0.18\sqrt{\frac{386\text{ in./s}^2}{0.353\text{ in.} + 0.244\text{ in.} + 0.0397\text{ in.}}} \\ &= 4.43\text{ Hz}\end{aligned}$$

Predicted Peak Acceleration

The peak accelerations due to the 1st, 2nd and 3rd harmonics of the dynamic force, as required in Table 5-2, are determined from Equation 5-2 with w_t = (4.00 psf + 56.4 psf + 4.20 psf) + 5.40 psf for steel framing = 70.0 psf, w_p = 4.2 psf, β = 0.060, i = 1, 2, 3 and with α_1 = 1.50, α_2 = 0.60 and α_3 = 0.10 from Table 5-2. The predicted peak accelerations for step frequencies from 2.0 to 2.75 Hz using Equations 5-1 and 5-2 are shown in Table 5-4 and plotted in Figure 5-4. The maximum predicted acceleration, 40.1%g, occurs when the step frequency is 2.23 Hz, for which the second harmonic frequency approximately equals the natural frequency, resulting in resonance.

Evaluation

The maximum peak acceleration of 40.1%g, which far exceeds the recommended tolerance limit of 4 to 7%g, indicates that the vibrations will be unacceptable, not only for the aerobics floor, but also for adjacent areas on the second floor. Furthermore, other areas of the building supported by the aerobics floor columns will probably be subjected to vertical accelerations that are unacceptable for most occupancies.

The floor framing shown in Figure 5-3 should not be used for aerobic activities. For an acceptable structural system, the natural frequency of the structural system needs to be substantially increased. Significant increases in the stiffnesses of both the beams and the girders are required. A lightweight concrete masonry (CMU) wall over the girder may be a cost effective means for stiffening the girder.

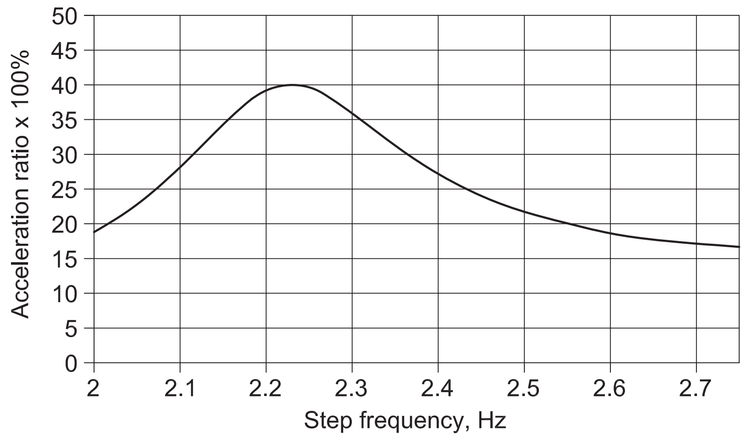


Fig. 5-4. Peak acceleration versus step frequency results for Example 5.2.

Chapter 6
Design for Sensitive Equipment and Sensitive Occupancies

This chapter provides guidance for evaluation of vibrations of floors supporting sensitive equipment, such as precision imaging, measurement, manufacturing instruments, and floors supporting sensitive occupancies such as hospital patient rooms and operating rooms. The vibration tolerance limits for sensitive equipment are usually much more stringent than the limits for human comfort. In relation to human comfort, often only vibrations at relatively low frequencies—up to about 9 Hz—need to be considered, but for sensitive equipment and occupancies, vibrations at higher frequencies also need to be considered.

6.1 EVALUATION OF VIBRATIONS IN AREAS WITH SENSITIVE EQUIPMENT

6.1.1 Tolerance Limits

The tolerance limits relating to human comfort typically are stated in terms of sinusoidal acceleration or velocity at a single frequency. In contrast, the suppliers of sensitive equipment often provide specific tolerance limits in terms of (1) peak (zero-to-peak) or peak-to-peak velocity or acceleration, (2) narrowband spectral velocity or acceleration, or (3) one-third octave spectral velocity or acceleration. Floor evaluations and designs should be based on specific limits for the equipment items of concern when possible. However, if the equipment items or their tolerances are not known, it has become typical practice to rely on generic tolerance limits for specifying the required vibration performance of floors.

Section 6.1.4 provides an evaluation procedure against generic velocity limits and Section 6.1.5 against specific tolerance limits.

6.1.2 Modal Parameters and Mode Shape Scaling

The response prediction methods in this chapter are based on the fundamental mode of the floor. Thus, for a given floor structure, one needs estimates of the natural frequency, modal mass or effective weight, mode shape, and damping of that mode.

The fundamental frequency used in the present chapter is somewhat different from that given in Chapter 3; here it is taken as the minimum of the beam mode and girder mode fundamental frequencies (Liu, 2015):

$$f_n = \min(f_b, f_g) \tag{6-1}$$

where the beam and girder natural frequencies, f_b and f_g, are computed using Equation 3-1 or 3-3.

The effective weight, W, may be obtained from Equation 4-5, or rational analysis and damping ratios may be estimated by use of Table 4-2.

The unity-normalized mode shape, which has the value of 1.0 at midbay, is estimated using the following, which describe single-curvature, two-way bending within the bay.

$$\phi = \sin\left(\frac{\pi x}{L_b}\right)\sin\left[\frac{\pi\left(y+L_g\right)}{3L_g}\right] \text{ if } f_b \le f_g \tag{6-2a}$$

$$\phi = \sin\left[\frac{\pi\left(x+L_b\right)}{3L_b}\right]\sin\left(\frac{\pi y}{L_g}\right) \text{ if } f_b > f_g \tag{6-2b}$$

where x, y, L_b and L_g are defined as shown in Figure 6-1. Note that the x-direction is always parallel to the beam spans. Alternatively, the mode shapes may be determined by finite element methods or other computational means.

When the equipment location and/or the walker location are not at midbay, the vibration response at the equipment location may be found by multiplying the midbay response by the mode shape value at the location of interest, ϕ_E, and the mode shape value at the walker location, ϕ_W, as determined from Equation 6-2a or 6-2b or from finite element analysis.

6.1.3 Conceptual Models of Floor Vibrations Due to Footfalls

Floor vibrations due to walking result from a series of footfall impulses; typical oscillatory motion is illustrated in Figures 1-4(a) and 1-8(a).

Severe vibrations can build up in a floor if it is subjected to a force at its natural frequency causing a resonant build-up such as the one shown in Figure 1-8(a). Because a walking person exerts significant dynamic forces only in the first four harmonics, as evident from the dynamic coefficients in Table 1-1, only low-frequency floors—floors with natural frequencies below the fourth harmonic maximum frequency—can experience such resonant build-ups due to walking.

In high-frequency floors—floors with all natural frequencies above the fourth harmonic maximum frequency— several cycles of vibration at the floor's natural frequency occur between successive impacts. A typical high-frequency floor

Table 6-1. Walking Parameters					
Walking Speed	**f_{step}, Hz**	**f_{4max}, Hz**	**Intermediate Zone Boundaries**		**γ**
			f_L, Hz	**f_U, Hz**	
Very slow	1.25	—	—	—	—
Slow	1.60	6.8	6	8	0.10
Moderate	1.85	8.0	7	9	0.09
Fast	2.10	8.8	8	10	0.08

f_L = intermediate zone lower boundary, Hz
f_U = intermediate zone upper boundary, Hz
f_{4max} = fourth harmonic maximum frequency, Hz

response to walking is shown in Figure 1-4(a). In this case, it is appropriate to consider each footstep as exerting an impulse, resulting in a vibration that rapidly reaches a peak and then decays until arrival of the next footstep.

Floors with natural frequencies in an intermediate zone around the fourth harmonic maximum frequency probably exhibit behavior between that of low- and high-frequency floors.

Table 6-1 includes the four walking speeds used in this chapter. Very slow walking applies to areas with one or two walkers and limited walking paths; examples are laboratories with fewer than three workers and medical imaging rooms. Slow walking applies to areas with three or four potential walkers and limited walking paths. Moderate walking applies to busy areas with fairly clear walking paths. Fast walking applies to areas with clear walking paths, such as corridors.

Table 6-1 includes the average step frequency, f_{step}, the fourth harmonic maximum frequency, f_{4max}, and the intermediate zone lower and upper boundaries for each speed. A dynamic load parameter, γ, based on the Willford et al. (2007) second through fourth dynamic coefficients in Table 1-1, is also included. Note that the response to very slow walking is computed using the impulse response equations in the following sections, thus f_{4max}, f_L, f_U and γ are not required.

6.1.4 Evaluation Against Generic Velocity Limits

The generic tolerance limits are given in terms of root-mean-square (RMS) spectral velocities in one-third octave bands of frequency. These limits, typically labeled as vibration criteria (VC) curves, are shown in Figure 6-2; Table 6-2 presents a list of equipment and activities to which the generic

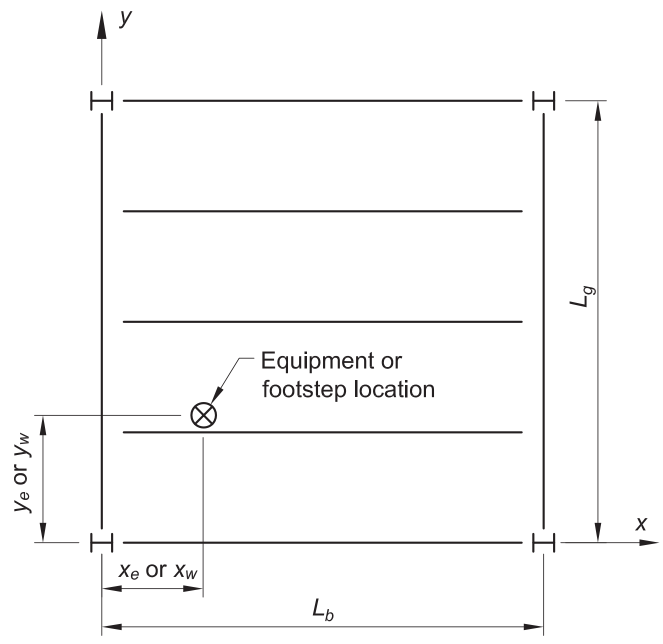


Fig. 6-1. Bay showing equipment and footstep locations.

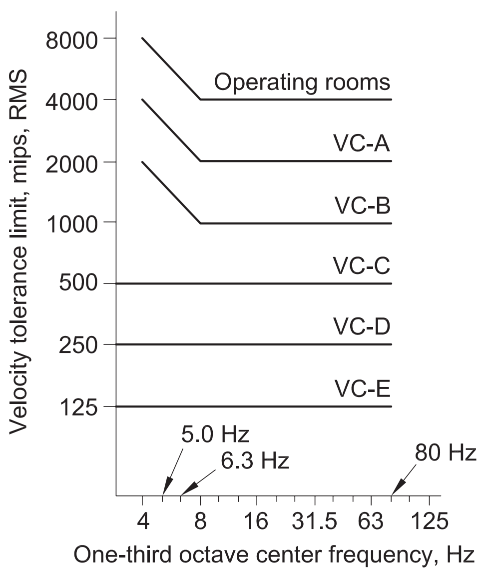


Fig. 6-2. Generic vibration criteria (VC) curves.

Table 6-2. Generic Vibration Criteria Tolerance Limits		
Designation	**Tolerance Limit[1], mips**	**Applicability**
—	32,000	Ordinary workshops[2]
—	16,000	Offices[2]
—	8,000	Computer equipment Residences[2,3]
—	6,000	Hospital patient rooms[4]
—	4,000	Surgery facilities, laboratory robots Bench microscopes up to 100×, operating rooms[5]
VC-A	2,000	Microbalances, optical comparators, mass spectrometers Industrial metrology laboratories, spectrophotometers Bench microscopes up to 400×
VC-B	1,000	Microsurgery, microtomes and cryotomes for 5 to10 μm slices Tissue and cell cultures, optical equipment on isolation tables Bench microscopes at greater than 400×, atomic force microscopes
VC-C	500	High-precision balances, spectrophotometers, magnetic resonance imagers Microtomes and cryotomes for <5 μm slices, chemotaxis Electron microscopes at up to 30,000×
VC-D	250	Cell implant equipment, micromanipulation Confocal microscopes, high-resolution mass spectrometers Electron microscopes (SEMs, TEMs) at greater than 30,000×
VC-E	125	Unisolated optical research systems, extraordinarily sensitive systems

[1] As measured in one-third octave bands over the frequency range 8 to 80 Hz (VC-A and VC-B) or 1 to 80 Hz (VC-C through VC-E); see Figure 6-2.
[2] Provided for reference only. Evaluate using Chapter 4 or Chapter 7.
[3] Corresponds to approximate average threshold of perception (ASA, 1983).
[4] When required by FGI (2014). Evaluate using Section 6.2.
[5] Corresponds to approximate threshold of perception of most sensitive humans (ASA, 1983). Evaluate using Section 6.2.

limits apply (Ungar et al., 2006, adapted from Amick et al., 2005). The velocity tolerance unit is micro-in./s (mips). The curves are valid below 80 Hz, but the prediction methods in this chapter have only been verified to approximately 15 Hz.

In terms of these velocities, the root-mean-square (RMS) floor vibrations at midbay due to walking at midbay may be estimated using Equation 6-3a for very slow walking and Equation 6-3b for slow, moderate or fast walking. Equation 6-3a predicts the impulse response. The first expression in Equation 6-3b predicts the resonant response of low-frequency floors, and the second expression predicts the impulse response of high-frequency floors. In the intermediate zone, the response is predicted using linear interpolation between the resonant response at f_L and impulse response at f_U.

$$V_{1/3} = \frac{250\times10^6}{\beta W}\frac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-3a}$$

$$V_{1/3} = \left|\begin{array}{l} \dfrac{175\times10^6}{\beta W\sqrt{f_n}}e^{-\gamma f_n} \text{ if } f_n \le f_L \\ \dfrac{250\times10^6}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right)\text{if } f_n \ge f_U \end{array}\right. \tag{6-3b}$$

where γ, f_L, f_U and f_{step} are from Table 6-1 and W is the effective weight in lb.

The resonant response expression is based on Liu (2015) and pertains to a six-footstep resonant build-up. The impulse response expression is based on Liu and Davis (2015) and the effective impulse approach from Chapter 1; it considers a four-second series of footsteps. The expressions are for a 168-lb walker and a total walking event duration of 8 seconds and include an empirical adjustment based on measured data selected so that the calculated results are exceeded by only 10% of the measured data.

Figure 6-3 presents design aid plots of $V_{1/3}W$ versus f_n obtained from calculations based on Equation 6-3 for two values of damping and the four walking speeds of Table 6-1.

6.1.5 Evaluation Against Specific Tolerance Limits

As mentioned in Section 6.1.1, limits for sensitive equipment are often expressed in other than generic terms; evaluation against these limits is addressed in this section. The prediction equations presented here are based on the references and assumptions cited after Equation 6-3. All of these equations pertain to midbay vibrations due to walking at midbay; mode shape scaling can be used to obtain estimates for other walking and vibration observation locations.

The design aid plots in this section are for the four walking speeds of Table 6-1 and, where applicable, for damping ratios of 0.03 and 0.05.

Waveform Peak Velocity or Acceleration Specific Limit

Figure 6-4 shows a typical acceleration waveform associated with walking, identifies the record's peak value, a_p, and illustrates a peak acceleration tolerance limit, $a_{p,Lim}$.

The peak velocity, v_p, (sometimes called the zero-to-peak velocity) in mips may be estimated from Equation 6-4, and the peak acceleration, a_p, may be estimated from Equation 6-5. Equations 6-4a and 6-5a apply for very slow walking, and Equations 6-4b and 6-5b apply for slow, moderate and fast walking. When the natural frequency is lower than the fourth harmonic maximum frequency, f_{4max}, from Table 6-1, the maximum of the resonant response (the first expression) and the impulse response (the second expression) is used in Equations 6-4b and 6-5b. When the natural frequency is f_{4max} or higher, the impulse response is used. The peak-to-peak values may be taken as twice the peak zero-to-peak values. Figures 6-5 and 6-6 are design aids for the evaluation of Equations 6-4 and 6-5.

$$v_p = \frac{19\times10^9}{W}\frac{f_{step}^{1.43}}{f_n^{1.3}} \tag{6-4a}$$

$$v_p = \max\left|\begin{array}{l} \dfrac{1.3\times10^9}{\beta W f_n}e^{-\gamma f_n} \text{ if } f_n \le f_{4max} \\ \dfrac{19\times10^9}{W}\dfrac{f_{step}^{1.43}}{f_n^{1.3}} \end{array}\right. \tag{6-4b}$$

$$\frac{a_p}{g} = \frac{310}{W}\frac{f_{step}^{1.43}}{f_n^{0.3}} \tag{6-5a}$$

$$\frac{a_p}{g} = \max\left|\begin{array}{l} \dfrac{22}{\beta W}e^{-\gamma f_n} \text{ if } f_n \le f_{4max} \\ \dfrac{310}{W}\dfrac{f_{step}^{1.43}}{f_n^{0.3}} \end{array}\right. \tag{6-5b}$$

where γ, f_{4max} and f_{step} are from Table 6-1 and W is the effective weight in lb.

Narrowband Spectral Velocity or Acceleration Specific Limit

Figure 6-7 shows the narrowband acceleration spectrum corresponding to the acceleration waveform in Figure 6-4. It identifies the maximum value, A_{NB}, and includes a curve corresponding to a specific illustrative limit, $A_{NB,Lim}$.

The greatest narrowband spectral velocity, V_{NB}, and the greatest narrowband spectral acceleration, A_{NB}, occur at the natural frequency of the floor. The former may be estimated (in mips) from Equation 6-6 and the latter from Equation 6-7. Equations 6-6a and 6-7a apply for very slow walking. Equations 6-6b and 6-7b apply for slow, moderate

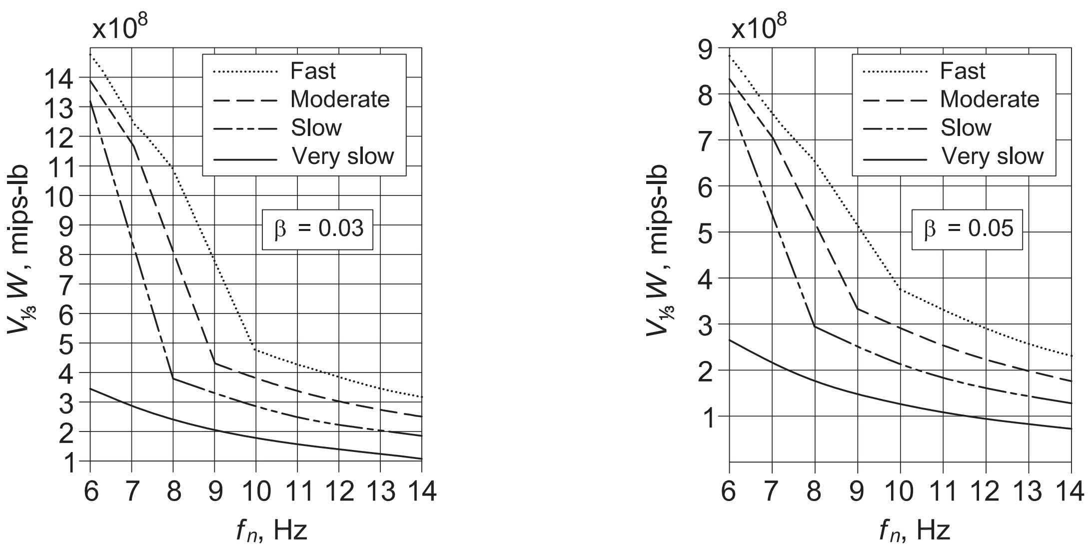


Fig. 6-3. One-third octave spectral velocity design aid (Equation 6-3).

and fast walking; the first expression predicts the resonant response of low-frequency floors, and the second expression predicts the impulse response of high-frequency floors. The response of a floor with fundamental natural frequency between f_L and f_U (Table 6-1) is predicted using linear interpolation between the resonant response at f_L and the impulse response at f_U. Figures 6-8 and 6-9 are design aids for evaluating Equations 6-6 and 6-7.

$$V_{NB} = \frac{490\times10^6}{\beta W}\frac{f_{step}^{2.43}}{f_n^{2.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-6a}$$

$$V_{NB} = \begin{cases} \dfrac{440\times10^6}{\beta W f_n}e^{-\gamma f_n} & \text{if } f_n \le f_L \\ \dfrac{490\times10^6}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{2.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) & \text{if } f_n \ge f_U \end{cases} \tag{6-6b}$$

$$\frac{A_{NB}}{g} = \frac{8.0}{\beta W}\frac{f_{step}^{2.43}}{f_n^{1.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-7a}$$

$$\frac{A_{NB}}{g} = \begin{cases} \dfrac{7.2}{\beta W}e^{-\gamma f_n} & \text{if } f_n \le f_L \\ \dfrac{8.0}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{1.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) & \text{if } f_n \ge f_U \end{cases} \tag{6-7b}$$

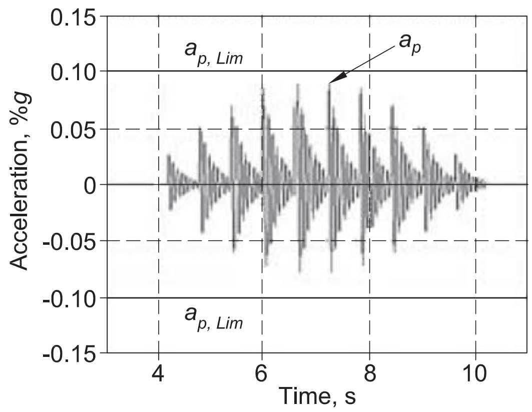


Fig. 6-4. Example acceleration waveform and peak acceleration tolerance limit.

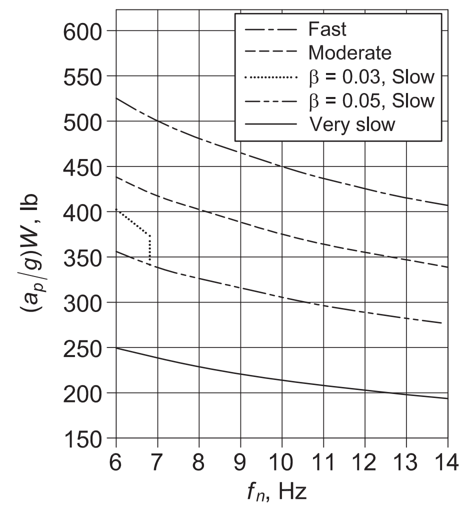


Fig. 6-6. Waveform peak acceleration design aid (Equation 6-5).

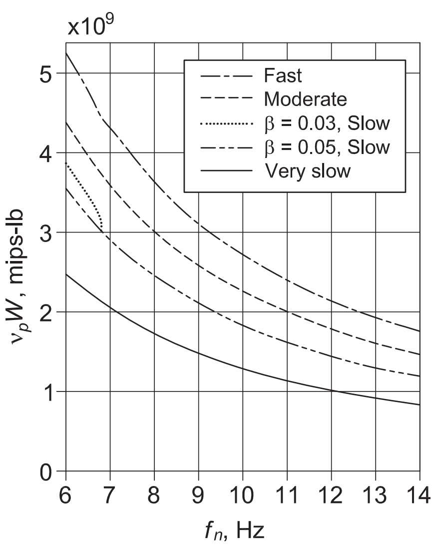


Fig. 6-5. Waveform peak velocity design aid (Equation 6-4).

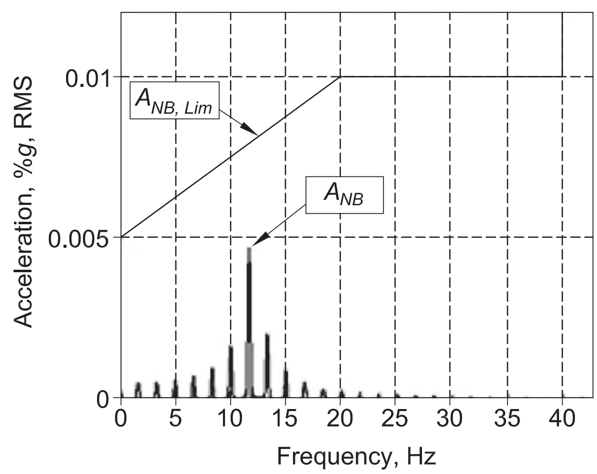


Fig. 6-7. Example narrowband acceleration spectrum and tolerance limit.

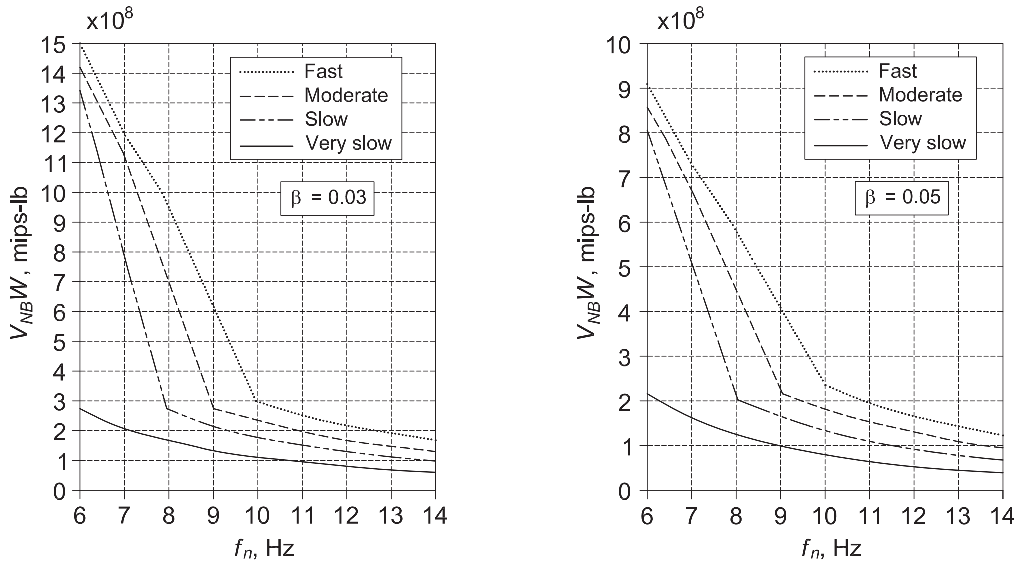


Fig. 6-8. Narrowband spectral velocity design aid (Equation 6-6).

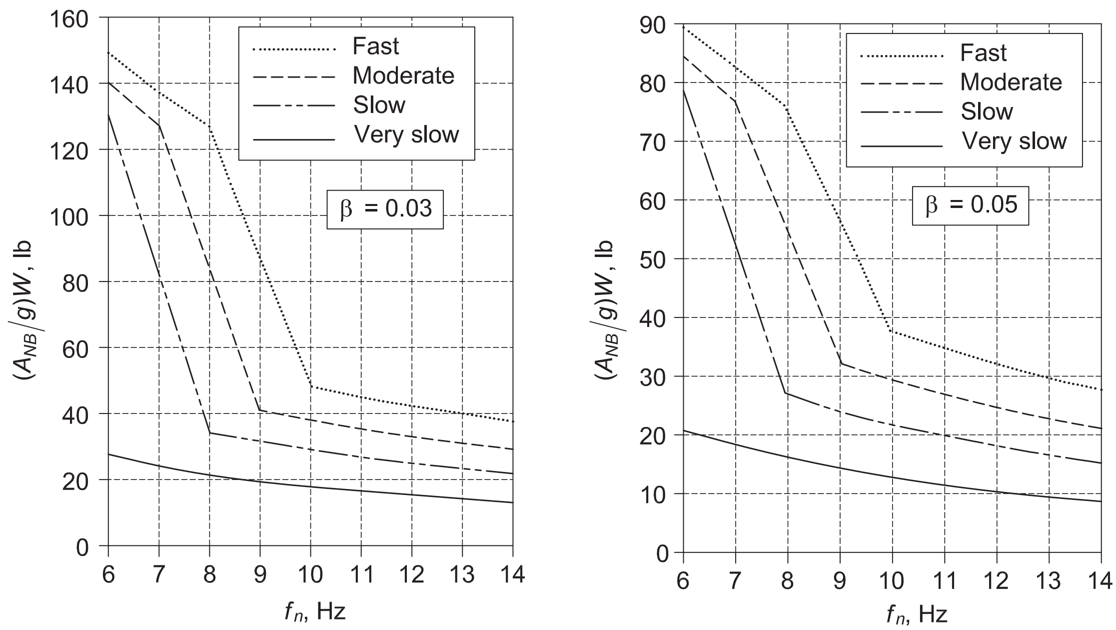


Fig. 6-9. Narrowband spectral acceleration design aid (Equation 6-7).

where γ, f_L, f_U and f_{step} are from Table 6-1 and W is the effective weight in lb.

One-Third Octave Spectral Velocity or Acceleration Specific Limit

Figure 6-10 shows the one-third octave band spectrum of velocity corresponding to the walking event shown in Figure 6-4 (narrowband spectral velocity is shown dashed), and also an illustrative one-third octave band limit curve, $V_{1/3,Lim}$.

The greatest spectral velocity, $V_{1/3}$, and acceleration, $A_{1/3}$, occur in the one-third octave band that includes the natural frequency of the floor. The one-third octave band spectral velocity, $V_{1/3}$, may be estimated by use of Equation 6-3, which is used for evaluation against generic velocity limits as described in Section 6.1.4.

The one-third octave band spectral acceleration, $A_{1/3}$, may be estimated by use of Equation 6-8a for very slow walking or Equation 6-8b for slow, moderate and fast walking. Linear interpolation is used between the resonant response at f_L and the impulse response at f_U in Equation 6-8b. Figure 6-11 is a design aid for evaluating Equation 6-8.

$$\frac{A_{1/3}}{g} = \frac{4.2}{\beta W} \frac{f_{step}^{2.43}}{f_n^{0.8}} \left(1 - e^{-2\pi\beta f_n / f_{step}}\right) \tag{6-8a}$$

$$\frac{A_{1/3}}{g} = \left| \begin{array}{l} \dfrac{6.4}{\beta W} e^{-\gamma f_n} \text{ if } f_n \le f_L \\ \dfrac{4.2}{\beta W} \dfrac{f_{step}^{2.43}}{f_n^{0.8}} \left(1 - e^{-2\pi\beta f_n / f_{step}}\right) \text{if } f_n \ge f_U \end{array} \right. \tag{6-8b}$$

where γ, f_L, f_U and f_{step} are from Table 6-1 and W is the effective weight in lb.

6.1.6 Effects of Floor Structure Parameter Changes

The preceeding prediction equations indicate that all of the floor response measures, except the peak velocity and peak acceleration above f_L (Table 6-1), are inversely proportional to the damping ratio, β, thus doubling the damping results when halving all of these measures.

The equations also show that the response measures are inversely proportional to the effective weight of the floor, W, and to its natural frequency, f_n, raised to some power. Because the natural frequency is proportional to $\sqrt{k/W}$, where k represents the stiffness of the floor, one finds that the various responses are inversely proportional to the factors in Table 6-3.

For example, increasing the effective weight by 60%—that is, by a factor of 1.6—would decrease $V_{1/3}$ at frequencies

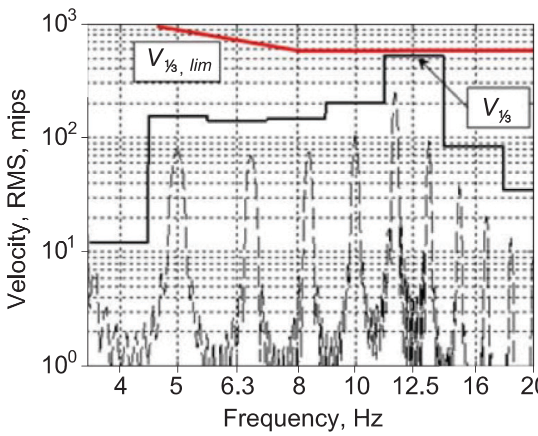


Fig. 6-10. Example one-third octave velocity spectrum and tolerance limit (narrowband velocity spectrum shown dashed).

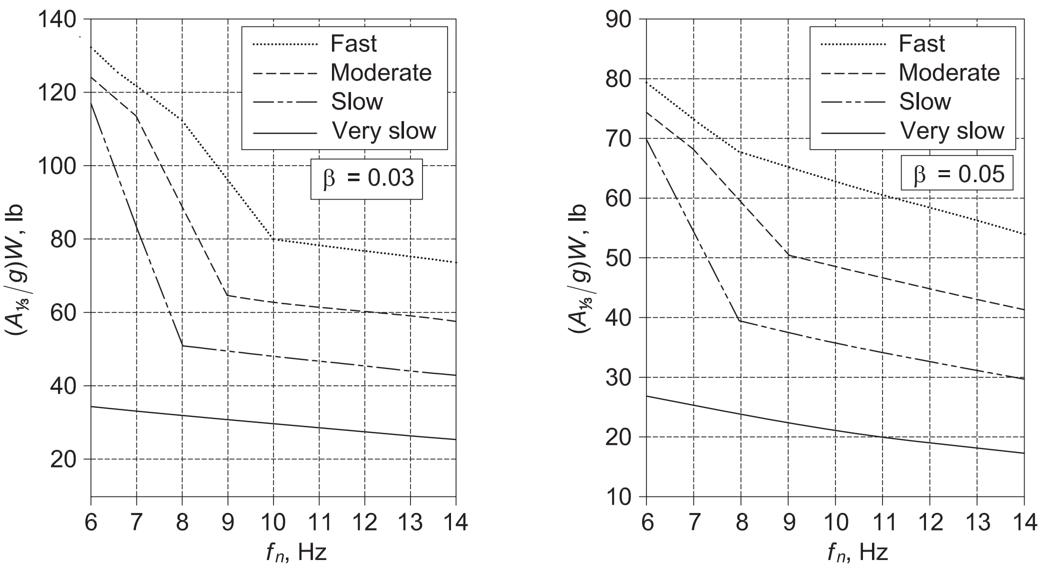


Fig. 6-11. One-third octave spectral acceleration design aid (Equation 6-8).

Table 6-3. Variation of Response Measures with Effective Weight and Stiffness						
Response Measure	$V_{1/3}$	$A_{1/3}$	v_p	a_p	V_{NB}	A_{NB}
Response < f_L	$W^{0.75}$, $k^{0.25}$	$W^{0.35}$, $k^{0.65}$	$W^{0.5}$, $k^{0.5}$	W	$W^{0.5}$, $k^{0.5}$	W
Response > f_U	$W^{0.1}$, $k^{0.9}$	$W^{0.6}$, $k^{0.4}$	$W^{0.35}$, $k^{0.65}$	$W^{0.85}$, $k^{0.15}$	$W^{-0.15}$, $k^{1.15}$	$W^{0.35}$, $k^{0.65}$

below f_L by a factor of $1.6^{0.75} \approx 1.42$ and at frequencies above f_U by only $1.6^{0.1} \approx 1.05$. The same increase of the effective weight would decrease A_{NB} below f_L by a factor of 1.6 but would increase V_{NB} above f_U by a factor of $1.6^{0.15} \approx 1.07$. The effects of stiffness changes may be assessed similarly—where the exponent on k is greater than that on W. Stiffness changes have a greater effect than changes in weight by the same factor.

Note that the tabulated factors do not apply if a floor parameter change results in changing the natural frequency from below f_L to above f_U, or vice versa. Such changes result in large changes in the response, as evident in the design aid charts.

6.1.7 Nonstructural Approaches to Reducing Vibration of Equipment

The vibration exposure of sensitive equipment may be reduced by relocating the equipment to a location with low mode shape amplitude and, thus, relatively low vibrations, such as near columns or stiff girders, and away from busy corridors. It also is advantageous to locate corridors along column lines. Limiting walking speeds—e.g., by introducing turns or obstacles in extended corridors—can significantly reduce walking-induced vibrations.

Carpeting, rubber mats and the like do not reduce the footfall forces transmitted to the structural floor appreciably and thus are not useful for reduction of walking-induced vibrations. According to Galbraith and Barton (1970), who studied the effect of shoe and surface hardness, the variation from test-to-test using the same footwear and surface was as great as the variability between tests with different footwear and surface.

Items of sensitive equipment may also be provided with mechanical vibration isolation means, which are available from many specialty suppliers. Such means need to be selected and designed to reduce the vibrations that are transmitted from the floor to the item at the natural frequencies of the floor.

6.2 EVALUATION OF VIBRATION IN AREAS WITH SENSITIVE OCCUPANTS

Operating rooms and hospital patient rooms are often designed to only allow imperceptible vibrations due to walking (Ungar, 2007). For these occupancies, the tolerance limit is usually expressed as one-third octave spectral velocity, and thus the response to walking is computed using Equation 6-9a for very slow walking and Equation 6-9b for slow, moderate or fast walking. In Equation 6-9b, the first expression predicts the resonant response of low-frequency floors, and the second expression predicts the impulse response of high-frequency floors. The response of floors with natural frequencies between f_L and f_U is predicted using linear interpolation between the resonant response at f_L and impulse response at f_U. Each expression includes a calibration factor selected such that the predicted and measured responses are equal, on average. (The predicted velocities from Equation 6-9 are less than those from Equation 6-3, reflecting the subjective nature of human response to vibrations versus the strict adherence required for sensitive equipment.)

$$V_{1/3} = \frac{200\times10^6}{\beta W}\frac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \qquad \text{(6-9a)}$$

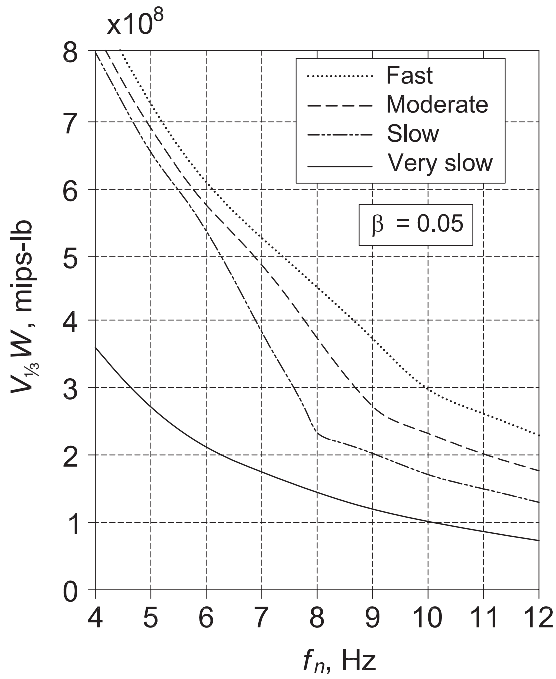


Fig. 6-12. Design aid for computing one-third octave band spectral velocity in areas with sensitive occupancies (Equation 6-9).

$$V_{1/3} = \begin{cases} \dfrac{120\times10^6}{\beta W\sqrt{f_n}}e^{-\gamma f_n} \text{ if } f_n \le f_L \\ \dfrac{200\times10^6}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \text{ if } f_n \ge f_U \end{cases} \tag{6-9b}$$

with linear interpolation between f_L and f_U, and where f_L, f_U, W, f_{step}, β and γ are determined as explained in Sections 6.1.2 and 6.1.3.

The natural frequency, f_n, is computed using Equation 6-1. Alternatively, it can be computed using the Chapter 7 finite element analysis methods that include the significant stiffening effect of partitions below and on the slab.

Equation 6-9 is plotted in Figure 6-12 for fast, moderate, slow and very slow walking. Most patient rooms and similar areas have full-height partitions; therefore, the curves are plotted for 5% damping.

The computed velocity response is at midbay due to walking at midbay, which is the most severe case. The response can be scaled by unity-normalized mode shape values as described in Section 6.1.2.

6.3 DESIGN EXAMPLES

Example 6.1—Evaluation of Framing Supporting Sensitive Equipment with a Generic Vibration Limit

Given:

Determine if the framing in Bay D-E/1-2 of Figure 6-13 will be satisfactory to support sensitive equipment with a generic vibration limit of 8,000 mips. The equipment can be anywhere in the bay. Fast walking can occur along the walking path shown. The slab is 5¼-in.-thick (total) concrete (w_c = 115 pcf, f'_c = 3 ksi), with 2-in. composite deck that weighs 2 psf. The superimposed dead and live loads are 4 psf and 11 psf, respectively, and there are several full-height partitions below the bay. The spandrel girder along Gridline 1 supports cladding. There are five bays in the east-west direction and two bays in the north-south direction. The beams and girders are ASTM A992 material.

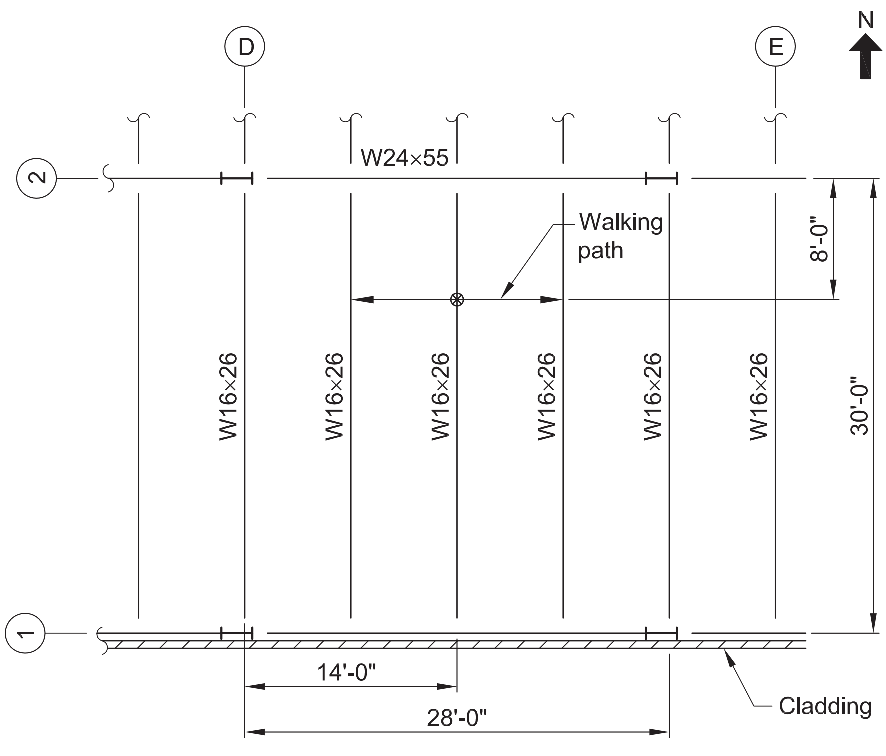


Fig. 6-13. Partial framing plan for Example 6.1.

Solution:

From the recommended values given in Table 4-2, the estimated damping ratio is determined as follows:

$$\begin{aligned}\beta &= 0.01(\text{structural system}) + 0.01\ (\text{ceiling and ductwork}) + 0.01\ (\text{similar to paper office fit-out}) + 0.02\ (\text{partitions}) \\ &= 0.05\end{aligned}$$

The properties of the deck are determined as follows:

$$\begin{aligned}E_c &= w^{1.5}\sqrt{f_c'} \\ &= (115\text{ pcf})^{1.5}\sqrt{3\text{ ksi}} \\ &= 2{,}140\text{ ksi}\end{aligned}$$

$$\begin{aligned}n &= \frac{E_s}{1.35E_c} \qquad (4\text{-}3c) \\ &= \frac{29{,}000\text{ ksi}}{1.35(2{,}140\text{ ksi})} \\ &= 10.0\end{aligned}$$

Total slab depth = 3.25 in. + 2.00 in. deck height

$$\begin{aligned}\text{Slab} + \text{deck weight} &= \frac{\left(3.25\text{ in.} + \dfrac{2.00\text{ in.}}{2}\right)(115\text{ pcf})}{12\text{ in./ft}} + 2.00\text{ psf} \\ &= 42.7\text{ psf}\end{aligned}$$

Beam Mode Properties

With an effective concrete slab width of 84.0 in. = 7.00 ft < $0.4L_j$ = 0.4(30.0 ft) = 12.0 ft, considering only the concrete above the steel form deck and using a dynamic concrete modulus of elasticity of $1.35E_c$, the transformed moment of inertia of the W16×26 beams is 1,110 in.^{4} (See Example 4.1 for example calculations.)

For each beam, the uniformly distributed load is

$$\begin{aligned}w_b &= (7.00\text{ ft})(11.0\text{ psf} + 42.7\text{ psf} + 4.00\text{ psf}) + 26.0\text{ plf} \\ &= 430\text{ plf}\end{aligned}$$

The corresponding deflection is

$$\begin{aligned}\Delta_b &= \frac{5w_bL_b^4}{384E_sI_b} \\ &= \frac{5(430\text{ plf})(30.0\text{ ft})^4(1{,}728\text{ in.}^3/\text{ft}^3)}{384(29\times10^6\text{ psi})(1{,}110\text{ in.}^4)} \\ &= 0.243\text{ in.}\end{aligned}$$

The beam mode fundamental frequency from Equation 3-3 is

$$f_b = 0.18\sqrt{\frac{g}{\Delta_b}} \tag{3-3}$$

$$= 0.18\sqrt{\frac{386 \text{ in./ s}^2}{0.243 \text{ in.}}}$$

$$= 7.17 \text{ Hz}$$

Using the procedures described in Chapter 4:

D_s = 7.63 in.^{4}/ft
D_b = 159 in.^{4}/ft
B_b = 28.1 ft
W_b = 77.7 kips

Girder Mode Properties

With an effective slab width of $0.4L_g$ = 0.4(28.0 ft) = 11.2 ft < 30.0 ft and considering the concrete in the deck ribs, the transformed moment of inertia for the W24×55 girders is 4,250 in.^{4} (See Example 4.1 for example calculations.)

For each girder, the equivalent uniform loading is

$$w_g = L_b\left(\frac{w_b}{S}\right) + \text{ girder weight per unit length}$$

$$= (30.0 \text{ ft})\left(\frac{430 \text{ plf}}{7.00 \text{ ft}}\right) + 55.0 \text{ plf}$$

$$= 1{,}900 \text{ plf}$$

The corresponding deflection is

$$\Delta_g = \frac{5w_gL_g^4}{384E_sI_g}$$

$$= \frac{5(1{,}900 \text{ plf})(28.0 \text{ ft})^4(1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{ psi})(4{,}250 \text{ in.}^4)}$$

$$= 0.213 \text{ in.}$$

From Equation 3-3, the girder mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} \tag{3-3}$$

$$= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.213 \text{ in.}}}$$

$$= 7.66 \text{ Hz}$$

Using the procedures in Chapter 4:

D_b = 159 in.^{4}/ft
D_g = 142 in.^{4}/ft
B_g = 40.0 ft
W_g = 70.9 kips

Combined Mode Properties

From Equation 6-1, the fundamental frequency is

$$\begin{aligned} f_n &= \min(f_b, f_g) \\ &= \min(7.17\text{ Hz}, 7.66\text{ Hz}) \\ &= 7.17\text{ Hz} \end{aligned} \tag{6-1}$$

From Equation 4-5, the equivalent combined mode panel weight is

$$\begin{aligned} W &= \frac{\Delta_b}{\Delta_b+\Delta_g}W_b + \frac{\Delta_g}{\Delta_b+\Delta_g}W_g \\ &= \frac{0.243\text{ in.}}{0.243\text{ in.}+0.213\text{ in.}}\left(77.7\text{ kips}\right) + \frac{0.213\text{ in.}}{0.243\text{ in.}+0.213\text{ in.}}\left(70.9\text{ kips}\right) \\ &= 74.5\text{ kips} \end{aligned} \qquad \text{(from Eq. 4-5)}$$

Predicted Response to Walking

From Table 6-1, $f_L = 8$ Hz and $\gamma = 0.08$ for fast walking. The tolerance limit is in terms of one-third octave velocity, and thus the response is predicted using Equation 6-3 for resonate build-up because $f_n < f_L = 8$ Hz.

$$\begin{aligned} V_{1/3} &= \frac{175\times10^6}{\beta W\sqrt{f_n}}e^{-\gamma f_n} \\ &= \frac{175\times10^6}{0.05\left(74{,}500\text{ lb}\right)\sqrt{7.17\text{ Hz}}}e^{-0.08\left(7.17\text{ Hz}\right)} \\ &= 9{,}890\text{ mips} \end{aligned} \qquad \text{(from Eq. 6-3b)}$$

For comparison, Figure 6-3 indicates $V_{1/3}W \approx 7.35\times10^8$ mips-lb for fast walking, $\beta = 0.05$, and $f_n = 7.17$ Hz. Thus, $V_{1/3} = 7.35\times10^8$ mips-lb/74,500 lb = 9,870 mips.

As shown in Figure 6-13, the walker is not at midbay, so the predicted response is scaled by the mode shape value at the walker location, ϕ_W. Because the beam frequency is lower than the girder frequency, Equation 6-2a applies. Taking D-2 as the origin of the coordinate system and noting that the x-direction is parallel to Gridline D, $x_W = 8$ ft and $y_W = 14$ ft (critical walker location is at midspan),

$$\begin{aligned} \phi_W &= \sin\left(\frac{\pi x_W}{L_b}\right)\sin\left[\frac{\pi\left(y_W+L_g\right)}{3L_g}\right] \\ &= \sin\left[\frac{\pi(8.00\text{ ft})}{30.0\text{ ft}}\right]\sin\left[\frac{\pi(14.0\text{ ft}+28.0\text{ ft})}{3(28.0\text{ ft})}\right] \\ &= 0.743 \end{aligned} \qquad \text{(from Eq. 6-2a)}$$

As discussed in Section 6.1.2, because the sensitive equipment can be anywhere in the bay, $\phi_W = 1.0$, and the predicted maximum velocity is

$$\begin{aligned} V_{1/3} &= 0.743(1.0)(9{,}890\text{ mips}) \\ &= 7{,}350\text{ mips} \end{aligned}$$

This velocity does not exceed the generic vibration limit of the sensitive equipment of 8,000 mips; therefore, the framing is considered satisfactory.

Example 6.2—Evaluation of Framing Supporting Sensitive Equipment with a Waveform Peak Acceleration Limit

Given:

Determine if the framing in Bay B-C/2-3 of Figure 6-14 is satisfactory to support sensitive equipment with a waveform peak acceleration limit of 0.1%g. The bay will be subjected to fast walking in the adjacent corridor. The slab is 8½-in.-thick (total) concrete (w_c = 145 pcf, f'_c = 3.5 ksi), with 2-in. composite deck that weighs 2 psf. The superimposed dead load is 9 psf and the live load is 11 psf. There are several full-height partitions supported by the bay. There are four bays in the east-west direction and four bays in the north-south direction. The beams and girders are ASTM A992 material.

Solution:

From the recommended values given in Table 4-2, the estimated damping ratio is determined as follows:

$$\begin{aligned}\beta &= 0.01 \text{ (structural system)} + 0.01 \text{ (ceiling and ductwork)} + 0.01 \text{ (similar to paper office fit-out)} + 0.02 \text{ (partitions)}\\ &= 0.05\end{aligned}$$

The properties of the deck are determined as follows:

$$\begin{aligned}E_c &= w^{1.5}\sqrt{f'_c}\\ &= \left(145 \text{ pcf}\right)^{1.5}\sqrt{3.5 \text{ ksi}}\\ &= 3{,}270 \text{ ksi}\end{aligned}$$

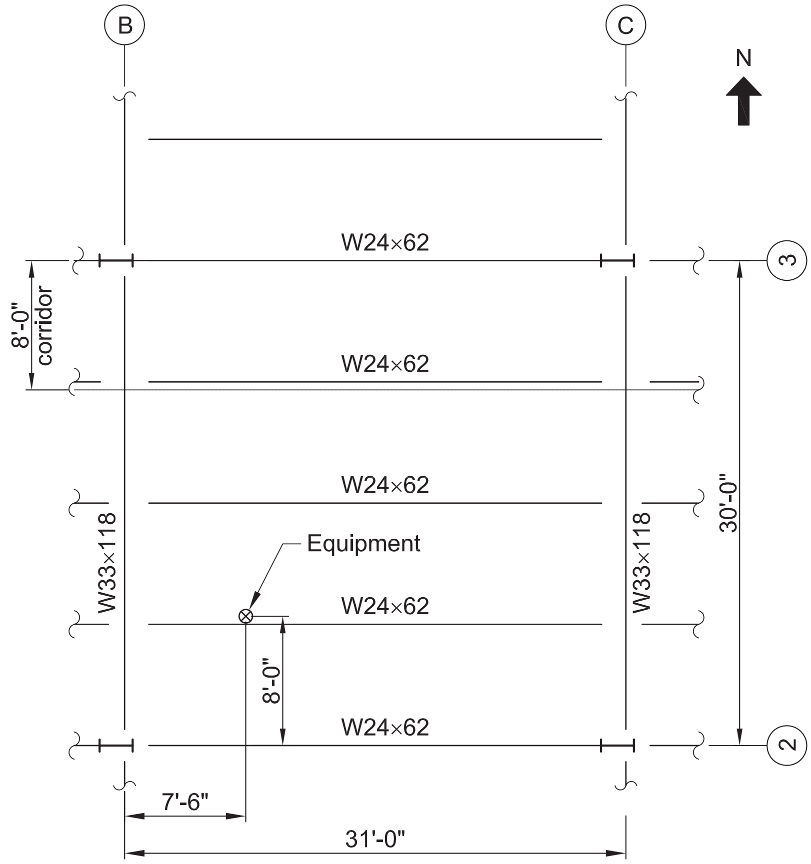


Fig. 6-14. Example 6.2 partial framing plan.

$$n = \frac{E_s}{1.35E_c} \tag{4-3c}$$
$$= \frac{29{,}000 \text{ ksi}}{1.35(3{,}270 \text{ ksi})}$$
$$= 6.57$$

Total slab depth = 6.50 in. + 2.00 in. deck height

$$\text{slab} + \text{deck weight} = \frac{\left(6.50 \text{ in.} + \dfrac{2.00 \text{ in.}}{2}\right)(145 \text{ pcf})}{12 \text{ in./ft}} + 2.00 \text{ psf}$$
$$= 92.6 \text{ psf}$$

Beam Mode Properties

With an effective concrete slab width of 90.0 in. = 7.50 ft < $0.4L_b$ = 0.4(31.0 ft) = 12.4 ft, considering only the concrete above the steel form deck, and using a dynamic concrete modulus of elasticity of $1.35E_c$, the transformed moment of inertia of the W24×62 beams is 6,280 in.^{4}

For each beam, the uniformly distributed loading is

$$w_b = (7.50 \text{ ft})(11.0 \text{ psf} + 92.6 \text{ psf} + 9.00 \text{ psf}) + 62.0 \text{ plf}$$
$$= 907 \text{ plf}$$

which includes 11 psf live load and 9 psf for mechanical/ceiling. The corresponding deflection is

$$\Delta_b = \frac{5w_bL_b^4}{384E_sI_b}$$
$$= \frac{5(907 \text{ plf})(31.0 \text{ ft})^4(1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{psi})(6{,}280 \text{ in.}^4)}$$
$$= 0.103 \text{ in.}$$

The beam mode fundamental frequency from Equation 3-3 is

$$f_b = 0.18\sqrt{\frac{g}{\Delta_b}} \tag{3-3}$$
$$= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.103 \text{ in.}}}$$
$$= 11.0 \text{ Hz}$$

Using the procedures described in Chapter 4:

B_b = 32.6 ft
D_s = 64.2 in.^{4}/ft
D_b = 841 in.^{4}/ft
W_b = 183 kips

Girder Mode Properties

With an effective slab width of $0.4L_g$ = 0.4(30.0 ft) = 12.0 ft < 31.0 ft and considering the concrete in the deck ribs, the transformed moment of inertia of the W33×118 girders is 19,500 in.^{4}

For each girder, the equivalent uniform loading is

$$w_g = L_b\left(\frac{w_b}{S}\right) + \text{girder weight per unit length}$$
$$= (31.0 \text{ ft})\left(\frac{907 \text{ plf}}{7.50 \text{ ft}}\right) + 118 \text{ plf}$$
$$= 3{,}870 \text{ plf}$$

The corresponding deflection is

$$\Delta_g = \frac{5w_g L_g^4}{384E_s I_g}$$
$$= \frac{5(3{,}870 \text{ plf})(30.0 \text{ ft})^4(1{,}728 \text{ in.}^3/\text{ft}^3)}{384(29\times10^6 \text{ psi})(19{,}500 \text{ in.}^4)}$$
$$= 0.125 \text{ in.}$$

From Equation 3-3, the girder mode fundamental frequency is

$$f_g = 0.18\sqrt{\frac{g}{\Delta_g}} \tag{3-3}$$
$$= 0.18\sqrt{\frac{386 \text{ in./s}^2}{0.125 \text{ in.}}}$$
$$= 10.0 \text{ Hz}$$

Using the procedures in Chapter 4:

$B_g = 58.0$ ft
$D_b = 842$ in.^{4}/ft
$D_g = 631$ in.^{4}/ft
$W_g = 217$ kips

Combined Mode Properties:

From Equation 6-1, the fundamental frequency is determined as follows:

$$f_n = \min(f_b, f_g) \tag{6-1}$$
$$= \min(11.0 \text{ Hz}, 10.0 \text{ Hz})$$
$$= 10.0 \text{ Hz}$$

From Equation 4-5, the equivalent mode panel weight is

$$W = \frac{\Delta_b}{\Delta_b + \Delta_g}W_b + \frac{\Delta_g}{\Delta_b + \Delta_g}W_g \qquad \text{(from Eq. 4-5)}$$
$$= \frac{0.103 \text{ in.}}{0.103 \text{ in.} + 0.125 \text{ in.}}(183 \text{ kips}) + \frac{0.125 \text{ in.}}{0.103 \text{ in.} + 0.125 \text{ in.}}(217 \text{ kips})$$
$$= 202 \text{ kips}$$

Unity-Normalized Mode Shape Values

Because the girder frequency is less than the beam frequency, from Equation 6-2b, at the equipment location with $x_E = 7.5$ ft and $y_E = 8$ ft:

$$\phi_E = \sin\left[\frac{\pi(x_E + L_b)}{3L_b}\right]\sin\left(\frac{\pi y_E}{L_g}\right) \qquad \text{(from Eq. 6-2b)}$$

$$= \sin\left[\frac{\pi(7.50\text{ ft} + 31.0\text{ ft})}{3(31.0\text{ ft})}\right]\sin\left[\frac{\pi(8.00\text{ ft})}{30.0\text{ ft}}\right]$$

$$= 0.716$$

With the walker location assumed to be at midspan of the beams, 1 ft-6 in. from the corridor wall, $x_W = 15.5$ ft and $y_W = 23.5$ ft,

$$\phi_W = \sin\left[\frac{\pi(x_W + L_b)}{3L_b}\right]\sin\left(\frac{\pi y_W}{L_g}\right) \qquad \text{(from Eq. 6-2b)}$$

$$= \sin\left[\frac{\pi(15.5\text{ ft} + 31.0\text{ ft})}{3(31.0\text{ ft})}\right]\sin\left[\frac{\pi(23.5\text{ ft})}{30.0\text{ ft}}\right]$$

$$= 0.629$$

Predicted Waveform Peak Acceleration

Because $f_n \geq f_{4max} = 8.8$ Hz, from Table 6-1 for fast walking, the second expression in Equation 6-5 is used with step frequency $f_{step} = 2.10$ Hz from Table 6-1 for fast walking. Because the walker and equipment are not at midbay, the response is scaled by the mode shape values.

$$\frac{a_p}{g} = \frac{310}{W}\frac{f_{step}^{1.43}}{f_n^{0.3}}\phi_E\phi_W \qquad \text{(from Eq. 6-5b)}$$

$$= \left(\frac{310}{202{,}000\text{ lb}}\right)\frac{(2.10\text{ Hz})^{1.43}}{(10.0\text{ Hz})^{0.3}}(0.716)(0.629)$$

$$= 0.00100$$

This acceleration ratio agrees with the a_p/g value obtained using Figure 6-6 multiplied by the mode shape values.

Because $a_p = 0.10\%g$ approximately equals the waveform peak acceleration limit, $0.1\%g$, the criterion is satisfied, and the framing is satisfactory to support the sensitive equipment.

Example 6.3—Evaluation of Framing Supporting Sensitive Equipment with Generic Vibration Criteria (VC) Limit

Given:

The floor of Example 6.2 is to be evaluated for its ability to support sensitive equipment with a VC-C limit (see Figure 6-2) due to very slow, slow, moderate and fast walking using the design aid shown in Figure 6-15.

From Table 6-2, the tolerance limit VC-C is a one-third octave spectral velocity of 500 mips. Figure 6-15 is Figure 6-3 with the $V_{1/3}W$ values at 10 Hz shown for very slow, slow, moderate and fast walking.

Solution:

From Example 6.2, $f_n = 10.0$ Hz, $W = 202$ kips, $\beta = 0.05$, and the unity-normalized mode shape values at the equipment and walker locations are 0.716 and 0.629, respectively.

From Figure 6-15, $V_{1/3}W = 1.3\times10^8$ mips-lb for very slow walking, and the predicted response is

$$V_{1/3} = \frac{1.3\times10^8 \text{ mips-lb}}{202{,}000 \text{ lb}}(0.716)(0.629)$$
$$= 290 \text{ mips}$$

which does not exceed the 500 mips tolerance limit.

From Figure 6-15, $V_{1/3}W = 2.1\times10^8$ mips-lb for slow walking, and the predicted response is

$$V_{1/3} = \frac{2.1\times10^8 \text{ mips-lb}}{202{,}000 \text{ lb}}(0.716)(0.629)$$
$$= 468 \text{ mips}$$

which does not exceed the 500 mips tolerance limit.

From Figure 6-15, $V_{1/3}W = 2.9\times10^8$ mips-lb for moderate walking, and the predicted response is

$$V_{1/3} = \frac{2.9\times10^8 \text{ mips-lb}}{202{,}000 \text{ lb}}(0.716)(0.629)$$
$$= 647 \text{ mips}$$

From Figure 6-15, $V_{1/3}W = 3.7\times10^8$ mips-lb for fast walking, and the predicted response is

$$V_{1/3} = \frac{3.7\times10^8 \text{ mips-lb}}{202{,}000 \text{ lb}}(0.716)(0.629)$$
$$= 825 \text{ mips}$$

The velocities for moderate and fast walking exceed the generic 500 mips VC-C tolerance limit, and therefore the framing is not satisfactory to support the sensitive equipment. If available, specific tolerance limits for specific items of equipment may permit greater vibrations and result in acceptability of the predicted magnitudes.

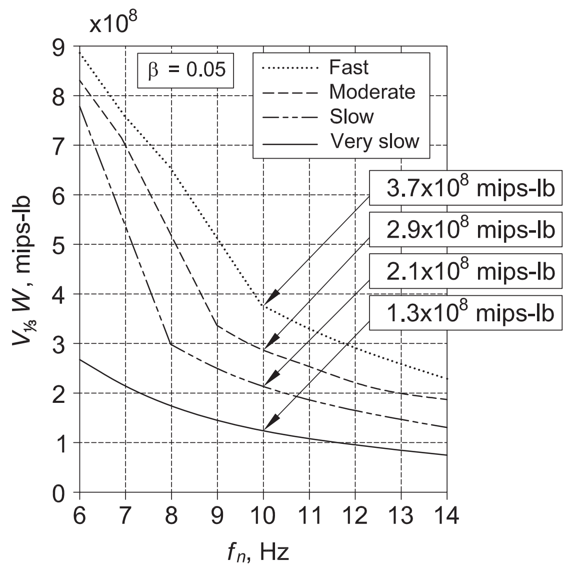


Fig. 6-15. One-third octave velocity design aid (Example 6.3).

Example 6.4—Evaluation of Framing Supporting Sensitive Hospital Patient Room

Given:

Determine if the framing in Bay D-E/1-2 from Example 6.1 will be satisfactory to support patient rooms with a sensitive occupancy tolerance limit of 6,000 mips. The patient bed can be anywhere in the bay; here it is assumed to be at midbay. Very slow walking can occur in the room, and fast walking can occur along the walking path shown in Figure 6-13.

Solution:

The following are the fundamental modal properties from Example 6.1:

Fundamental frequency: $f_n = 7.17$ Hz
Effective weight: $W = 74.5$ kips
Damping: $\beta = 0.05$
Unity-normalized mode shape at the corridor walking path: $\phi_W = 0.743$

Predicted Response to Walking

The bay is being evaluated for a sensitive occupancy. The response to very slow walking is predicted using Equation 6-9a, with $f_{step} = 1.25$ Hz from Table 6-1.

$$V_{1/3} = \frac{200\times10^6}{\beta W}\frac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-9a}$$

$$= \frac{200\times10^6}{(0.05)(74{,}500\text{ lb})}\frac{(1.25\text{ Hz})^{2.43}}{(7.17\text{ Hz})^{1.8}}\left(1-e^{-(2\pi)(0.05)(7.17\text{ Hz})/(1.25\text{ Hz})}\right)$$

$$= 2{,}220\text{ mips}$$

The response to fast walking in the corridor is predicted using Equation 6-9b for resonant build-up when $f_n < f_L = 8$ Hz, where f_L and γ are from Table 6-1.

$$V_{1/3} = \frac{120\times10^6}{\beta W\sqrt{f_n}}e^{-\gamma f_n} \qquad \text{(from Eq. 6-9b)}$$

$$= \frac{120\times10^6}{0.05\left(74{,}500\text{ lb}\right)\sqrt{7.17\text{ Hz}}}e^{-0.08\left(7.17\text{ Hz}\right)}$$

$$= 6{,}780\text{ mips}$$

As shown in Figure 6-13, because the walker is not at midbay, the predicted response is scaled by the mode shape value at the walker location.

Therefore, the predicted velocity with $\phi_W = 0.743$ is

$$V_{1/3} = 0.743(6{,}780\text{ mips})$$
$$= 5{,}040\text{ mips}$$

Evaluation

The predicted velocities do not exceed the 6,000 mips tolerance limit; therefore, objectionable vibrations are not expected, and the framing is satisfactory to support the sensitive-occupancy patient rooms. (For comparison, the predicted acceleration due to normal walking using the methods in Chapter 4 is 0.25%*g*.)

Chapter 7
Finite Element Analysis Methods

The methods recommended in Chapters 3 through 6 apply to typical structural systems with regular framing and uniform mass distributions, subject to loads described in those chapters. Examples of systems outside the scope of those chapters are cantilevered floors, floors supported by transfer girders, pedestrian bridges with flexible supports, switchback stairs with no intermediate landing support, and retrofit floors. Finite element analysis (FEA) can be used to predict vibration response for such systems. This chapter presents recommendations for the FEA of floors, pedestrian bridges and stairs subject to walking or running, floors and balconies subject to rhythmic activities, and floors supporting sensitive equipment, and includes examples.

7.1 INTRODUCTION

Vibration evaluation using FEA requires the same steps as those listed in the previous chapters. First, the part of the structure being evaluated is defined in terms of its geometry, mass, stiffness and damping. This step entails the development of a three-dimensional model appropriate for analyzing small-vibration amplitudes ranging from approximately 0.0002 in. to 0.05 in. Second, dynamic properties—natural frequencies and mode shapes—are predicted using eigenvalue analysis. Third, human-induced loads are represented by a Fourier series or effective impulse described in Chapter 1. Fourth, the response is computed and compared to the tolerance limit to predict whether or not vibrations will be objectionable.

For all loadings, the greatest response occurs if a loading harmonic frequency (i.e., an integer multiple of the step frequency) equals a natural frequency. However, the loading frequency must be in the expected step frequency range for the activity being evaluated. For instance, the range of step frequencies for walking is 1.6 to 2.2 Hz, and therefore, the loading frequency must be in this range.

FEA usually predicts numerous vibration modes, and it is sometimes difficult to visually identify responsive modes. The methods in this chapter overcome this problem by using the frequency response function (FRF), which is a plot of responsiveness, in units such as $\%g$/lb, versus forcing frequency. Resonant responses are predicted using the fast and efficient FRF method in which acceleration is the product of the FRF maximum magnitude and human-induced load. Time history analysis can be used instead, but it is often not practical because it is much more time consuming.

7.2 MODEL DEVELOPMENT

Extent of Model

Floor models should usually only include the floor, or a portion of the floor, containing the bay(s) being evaluated. Floors above or below should also be included if the evaluated floor is supported by a hanger from above or a column extending to a transfer girder below. The horizontal extent of the floor is determined using engineering judgment. Adjacent bays may restrain the bay being evaluated, which increases natural frequencies. They also may move synchronously with it, adding mass in motion and decreasing acceleration response. From field measurements, it has been found that floor motion is usually limited to several bays around the bay excited by human activity, in part because frictional damping causes energy loss over distance as well as time. Also, mass and stiffness vary from bay to bay even if the bays are nominally identical, so mode shapes usually only have a few bays in motion. In contrast, finite element analyses—especially of large floors with uniform framing and mass—often predict motion over unrealistically large areas, resulting in overprediction of the mass in motion and thus unconservative underprediction of acceleration response. Large areas of a floor may be included in the model, but if responsive modes extend over more than four or five bays, the extent of the model should be reduced. A simple approach is to include the bay being evaluated plus adjacent bays. For an interior floor bay evaluation, this results in a three-by-three grid of bays.

Balconies are similar to floors in that motion is often limited to portions of the balcony due to nonuniform mass or stiffness. The model can include large sections of the balcony, but should be reduced to only include seating sections between several adjacent raker beams if the predicted natural modes include motion over very large areas. Such a balcony is likely to have predicted mode shapes with unrealistically large areas. Even if the sections are nominally identical, there will be differences in reality.

Pedestrian bridge and stair models usually include the entire pedestrian bridge or stair, which will essentially vibrate as a beam. If the pedestrian bridge or stair is supported by flexible elements (e.g., a pedestrian bridge end connected to a spandrel girder), flexibility of those elements will decrease natural frequencies. This effect should be modeled by including the element, or the element plus a limited area such as a bay, in the model. Large areas of the supporting structure should not be included in the pedestrian bridge or stair model because the supporting structure has much larger mass, and unavoidable minor errors in its modeling can have a large effect on the predicted pedestrian bridge or stair response.

Slab Definition

It is recommended that slabs be modeled using orthotropic shell elements with flexural stiffnesses computed using basic mechanics techniques. The dynamic elastic modulus of the concrete should be computed as recommended in Chapter 3 ($1.35E_c$), and the shear modulus should be computed as $G_c = 1.35E_c/\left[2(1 + \nu)\right]$ where Poisson's ratio, ν, is 0.2. The slab flexural stiffness parallel to deck ribs is usually between two and five times higher than the stiffness perpendicular to the ribs. As indicated by Barrett (2006), the use of orthotropic stiffnesses, rather than equal stiffness in the two orthogonal directions, results in more accurate natural frequency and mode shape predictions.

Concrete cracks exist over girder lines in many slabs supported by steel deck, regardless of whether the construction is shored or unshored. However, experimentally measured natural frequencies and mode shapes are usually in reasonable agreement with FEA predictions using uncracked section properties (Barrett, 2006; Pavic et al., 2007; Davis, 2008). This is due in part to the very small displacement magnitudes associated with walking-induced vibrations and because the major cracks run parallel to the girder span and do not affect joist or girder stiffness. Also, the writers are not aware of any experimentally verified modeling technique for taking these cracks into account. Thus, uncracked section properties are recommended.

The shell mesh should be fine enough to allow the first few natural frequencies to remain approximately unchanged with further mesh refinements. The mesh should not be overly refined because additional degrees-of-freedom cause much longer analysis times and natural frequency predictions are only moderately accurate to within ±5 to 10%, regardless of the sophistication of the model. The mesh is sufficiently refined if further refinements do not result in natural frequency changes larger than 0.05 to 0.1 Hz. Meshes of approximately 1⁄10 the bay size are usually adequate.

Framing Members

Framing members should be modeled using regular three-dimensional frame elements in the plane of the shells as shown in Figure 7-1. (Note that the model is of a fairly large floor area. Depending on the analysis results and engineering judgment, it might need to be reduced as previously described.) Beams, joists and girders connected to concrete slabs can be considered fully composite because the horizontal shear generated by human-induced loads is easily resisted by all commonly used connectors, including puddle welds, screws and shear studs. Girders and joist girders supporting open-web steel joists should be modeled as partially composite due to joist seat elastic deformation. Effective transformed moments of inertia should be computed using methods and assumptions recommended in Chapter 3. Note that because the slab moment-of-inertia about its own centroidal axis is already included in the shell stiffness, that inertia term is excluded from the member transformed moment of inertia.

Rolled beam and girder end connections should be considered continuous for vibration analyses, even when they are designed and detailed as shear connections (Barrett, 2006; Smith et al., 2007; Davis, 2008). This is justified

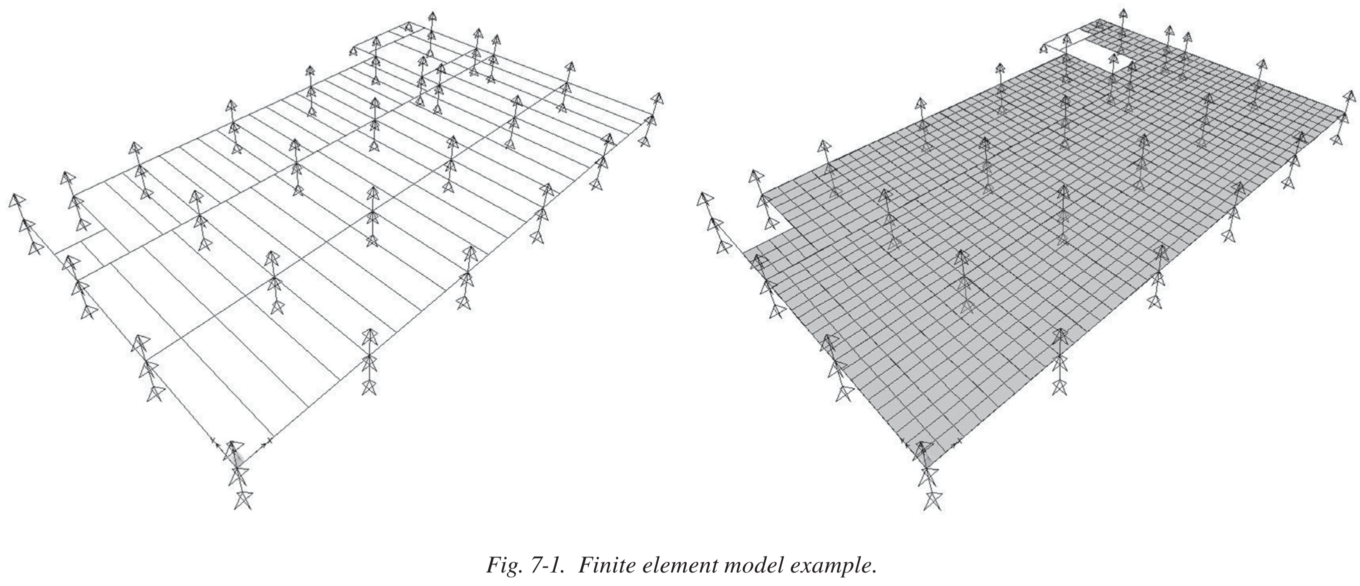


Fig. 7-1. Finite element model example.

because the very small member end moments generated by human-induced loads are easily resisted by bolt friction and the couple that forms between the slab and the bolts. Typical open-web joist and joist girder ends without extended bottom chords are hinged.

Spandrel beams and girders are usually significantly restrained by cladding. For light cladding, even with slip tracks, Barrett (2006) recommends increasing the effective moment of inertia by a factor of 2.5. This technique is conservative for spandrels supporting more substantial cladding such as masonry and is, therefore, recommended in general.

If columns are included in the model, they should be modeled using regular frame elements extending to the stories below and above. This extent is conservatively recommended because some natural modes will have single curvature column bending between stories.

Nonstructural Partitions Walls

Nonstructural partitions provide stiffness, which often significantly increases natural frequencies and affects mode shapes. Engineering judgment must be used to establish which walls should be included in the model. Partition stiffness should not be included if the walls are likely to be removed during future remodeling.

Typical drywall and steel stud partitions with vertical slip tracks behave as partially effective load-bearing walls during human-induced vibration. According to the research by Davis and Liu (2014), a vertical linear spring with 2.0 kip/in./ft of wall may be assigned to each slab shell node along each wall as shown in Figure 7-2. For example, if slab shell nodes are spaced 3 ft apart and the wall is below and above the slab (extending to a slip-track at the level above), a vertical spring with stiffness k = (2.0 kip/in./ft)(3 ft)(2 walls) = 12 kip/in. can be placed at each shell node along the wall.

Drywall and steel stud partitions supported on the slab, but not extending to the underside of the floor slab or the roof deck, probably behave as partially effective shear walls during human-induced vibration. However, limited experimental research has shown only slight changes in modal properties; therefore, it is recommended that the stiffness of such walls not be included in the model.

Concrete masonry or clay masonry partitions supported by the slab should be modeled as vertical shells with

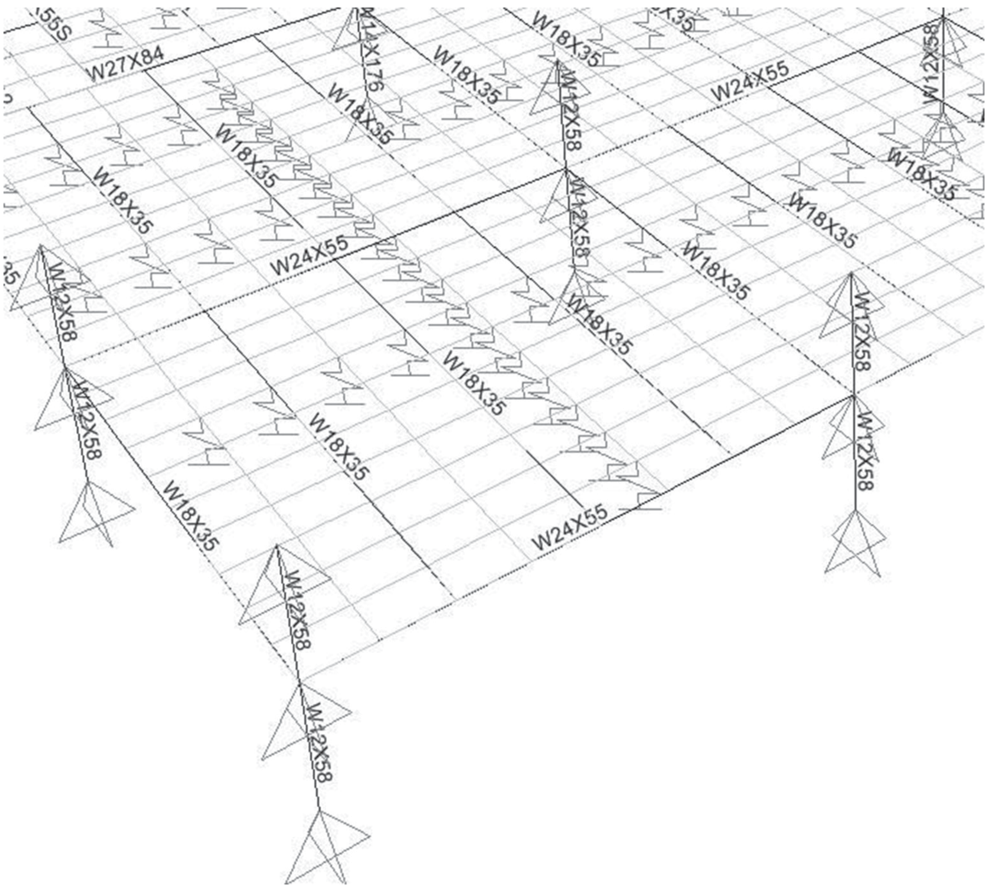


Fig. 7-2. Nonstructural partition springs added to Figure 7-1 example.

in-plane stiffness computed using basic mechanics and nominal material properties. Alternatively, a frame element with equivalent stiffness can be placed in the plane of the slab shells. These modeling techniques treat the wall as a bending or shear element. No experimentally verified guidelines are available for masonry partitions below the floor because the vertical slip connection details vary; however, these may be conservatively modeled as 2.0 kip/in./ft vertical springs along the wall.

Mass

Best estimates of the actual mass in place during the human-induced loading event must be used rather than masses corresponding to service or ultimate loads. Recommended actual in-place masses are given in Chapter 4. Structural analysis programs allow masses to be applied using various methods. Member masses should be computed by the program using material density and cross-sectional area. The resulting line mass is expressed as mass per unit length, such as plf-s^2/ft, which usually corresponds to the published lineal weight. Uniform mass of deck, concrete, superimposed dead load, and live load, in units such as psf-s^2/ft—depending on the features of the program—should be assigned to the slab shell elements. Supported partition wall masses can be applied to slab shell nodes along the wall, as line masses assigned to zero-stiffness frame elements in the plane of the slab, or as uniform mass applied to the slab shells.

Damping

As described in Chapter 4, damping is provided primarily by nonstructural elements such as ceilings, mechanical elements, furnishings and partitions. Damping probably varies spatially, but the effects of this variation are currently not predictable due to lack of research. Thus, the viscous damping ratio is estimated in the same manner as described in Chapters 4, 5 and 6, and assigned to each natural mode.

7.3 NATURAL FREQUENCIES AND MODES

Natural frequencies and mode shapes should be computed by solving the multiple degree-of-freedom undamped free-vibration eigenvalue problem, which is described in detail in vibrations textbooks (Clough and Penzien, 1993; Ewins, 2000; Chopra, 2011). The eigenvalue problem solution is valid for undamped and proportionally damped systems and can be used for floors (Barrett, 2006; Davis, 2008) and other structures covered by this Design Guide. Another option available in some programs is Ritz-vector analysis. Barrett (2006) reported that Ritz-vector analysis provided no advantage over eigenvalue analysis for floors in his research and resulted in illogical higher frequency mode predictions in some cases.

The number of modes to be computed by the program should be such that all modes with single curvature within a floor bay or pedestrian bridge span are included. Higher frequency modes with double curvature are usually not needed in vibration evaluation. A good guideline is to include modes with natural frequency up to approximately double the fundamental frequency. Example predicted mode shapes are shown in Figure 7-3. Note that automatic scaling of mode shapes sometimes obscures the shape; engineering judgment may be required to display a clear mode shape.

Observation of predicted natural frequencies and mode shapes is often insufficient to determine frequencies that, if matched by a dynamic load frequency, result in high responses. The first problem is that a large number of natural modes are often predicted. In the example shown in Figures 7-1, 7-2 and 7-3, there are 17 modes in the 6-Hz band

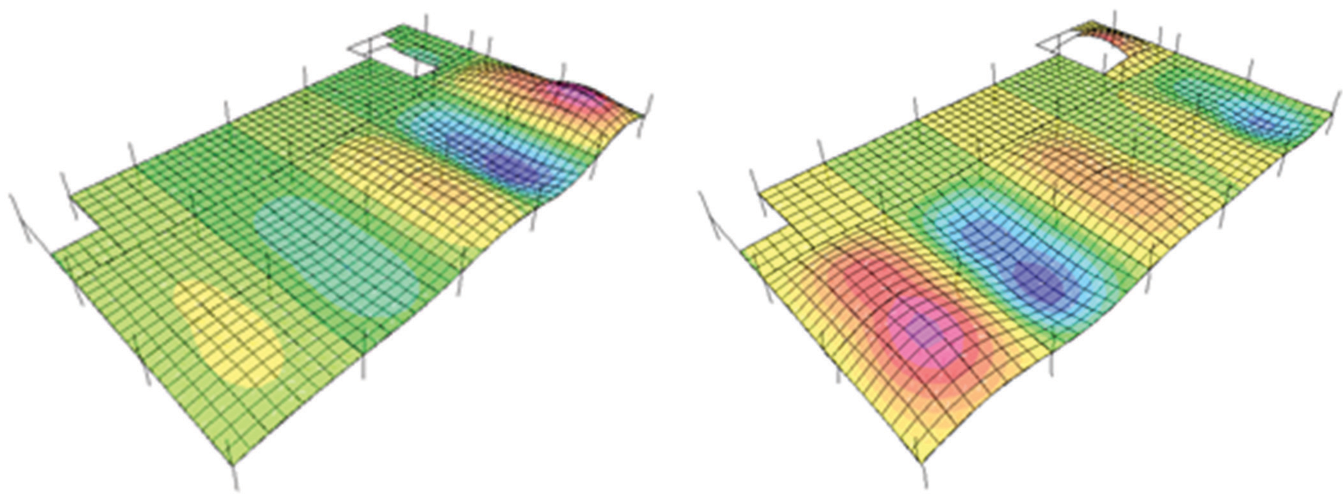


Fig. 7-3. Example predicted mode shapes for framing in Figure 7-1.

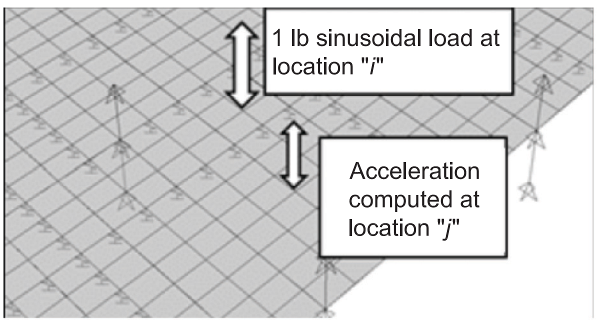
between the fundamental frequency and double the fundamental frequency. It is difficult to judge, through visual observation of the shapes, which of these modes will provide high response if excited. Also, multiple modes provide significant contributions in some cases, thus the highest computed response might occur at a frequency other than a natural frequency.

A frequency response function (FRF)—a plot of steady-state response due to sinusoidal load with unit amplitude versus frequency—is used to determine which mode(s) provides highest response, thus solving the problem. For the purposes of vibration serviceability evaluation, the FRF is constructed by: (1) defining the frequency band lower limit as 1 Hz below the fundamental frequency; (2) defining the upper limit as 1 Hz above the maximum computed modal frequency; (3) defining key frequencies within the band, usually at each modal frequency and several others depending on the desired resolution of the plot; and (4) computing and plotting the sinusoidal steady-state response at location j due to unit amplitude sinusoidal load at location i, with unit amplitude at each frequency within the band. (Note that FRFs can be computed for several response and load locations.) The process is illustrated in Figure 7-4. Several commercially available programs provide convenient features for computing the FRF magnitude. Other programs provide time history analysis, but not automatic calculation of FRFs. When using these, the FRF can be plotted by performing several time-history analyses and plotting steady-state accelerations.

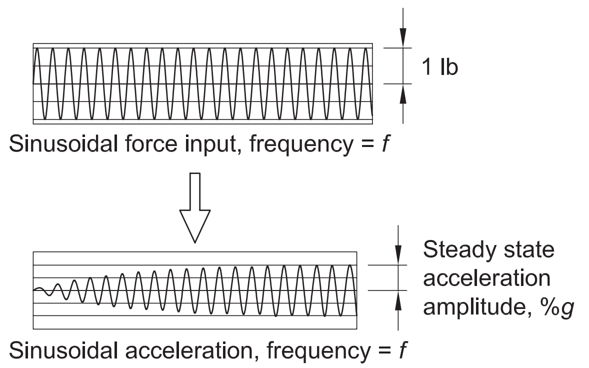
7.4 HUMAN COMFORT EVALUATION

7.4.1 Walking on Level Low- and High-Frequency Floors and Pedestrian Bridges

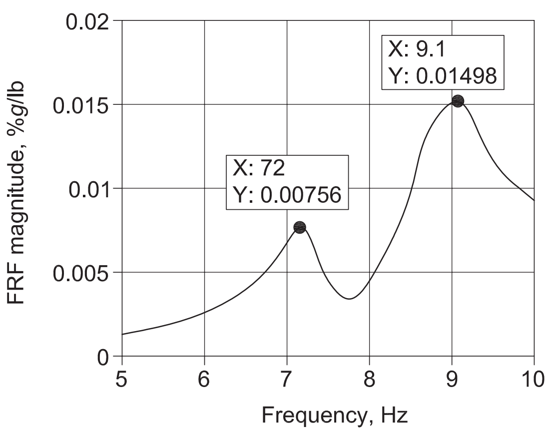
This section provides evaluation methods for level surfaces such as floors and pedestrian bridges subject to walking. A structure is unsatisfactory if the predicted sinusoidal peak acceleration exceeds the applicable human comfort tolerance limit from Figure 2-1. As described in Section 1.5, the maximum response of a low-frequency floor—having at least one natural frequency below 9 Hz—is due to a multiple-footstep resonant build-up, whereas the maximum response of a high-frequency floor is due to individual-footstep impulse response. Evaluation methods for low-frequency



(a) Load and acceleration response locations



(b) Sinusoidal load with unit amplitude and resulting response



(c) FRF magnitude plot

Fig. 7-4. Example predicted FRF for framing in Figure 7-1.

and high-frequency floors are presented in the following sections.

Low-Frequency Floors and Pedestrian Bridges

The resonant response can be predicted using the FRF method in which the predicted peak sinusoidal acceleration is the product of the maximum FRF magnitude, the force harmonic amplitude, and a partial resonant build-up factor (Davis, 2008; Davis and Murray, 2010).

The FRF magnitude is computed for vertical unit load at the walking load location and vertical acceleration at the affected occupant location—locations *i* and *j* in Figure 7-4(a). An average resonant build-up is approximately six footsteps, and the average stride length is approximately 2.5 ft, thus the average resonant build-up is due to footsteps along a path approximately 15 ft long. To evaluate the most severe likely vibration, engineering judgment should be used to identify an unobstructed walking path as close as possible to the maximum mode shape value. The walking load location should be set near the midlength of the walking path because this location has the average mode shape value along the walking path. If the affected occupant location is unknown, then it should be located as close as possible to the maximum mode shape value. For regular floor bays or pedestrian bridge spans, the walking load and affected occupant locations can usually be conservatively located at midbay or midspan. The frequency below 9 Hz, with maximum FRF magnitude, is referred to as the dominant frequency. However, this is not always a natural frequency, due to contributions from multiple modes, especially when damping is high and there are numerous closely spaced modes.

The predicted peak sinusoidal acceleration due to walking is

$$a_p = FRF_{Max}\alpha Q\rho \tag{7-1}$$

where

FRF_{Max} = maximum FRF magnitude at frequencies below 9 Hz, %g/lb
Q = bodyweight = 168 lb
α = dynamic coefficient
ρ = resonant build-up factor

The dynamic coefficient is computed using the following equation, which approximates the Willford et al. (2007) second through fourth harmonic dynamic coefficients in Table 1-1. The equation was derived using the procedure described in Section 2.2.1.

$$\alpha = 0.09e^{-0.075f_n} \tag{7-2}$$

where

f_n = dominant frequency, Hz

The following resonant build-up factor is for a six-footstep build-up and is derived from the envelope function for a viscously damped single-degree-of-freedom system subjected to sinusoidal load.

$$\rho = 50\beta + 0.25 \text{ if } \beta < 0.01 \tag{7-3a}$$

$$\rho = 12.5\beta + 0.625 \text{ if } 0.01 \le \beta < 0.03 \tag{7-3b}$$

$$\rho = 1.0 \text{ if } \beta \ge 0.03 \tag{7-3c}$$

where β is the viscous damping ratio from Chapter 4.

Pedestrian bridges subject to large groups should also be evaluated for group loads and lateral vibrations because synchronization of footsteps may cause very high accelerations and/or lateral bridge movement. If the predicted peak acceleration does not exceed the applicable tolerance limit from Figure 2-1, the bay or span is predicted to be satisfactory for vertical vibrations. See Section 4.2 for additional recommendations.

If the dominant frequency is below 3 Hz, the floor or pedestrian bridge will be vulnerable to vandal jumping (groups of people intentionally exciting the structure by jumping or bouncing in unison); because of this, the structure should also be evaluated for group rhythmic loads per Section 7.4.4.

High-Frequency Floors and Pedestrian Bridges

The peak acceleration due to an individual footstep is computed using the effective impulse method described in Section 1.5, based on the research by Willford et al. (2006) and Liu and Davis (2015). This method predicts the peak acceleration immediately after a footstep, a quantity that is not directly comparable to sinusoidal peak acceleration limits from Figure 2-1. Thus, it must be converted to equivalent sinusoidal peak acceleration (Davis et al., 2014).

The FRF magnitude is computed for unit load at the walking load location, *i*, and acceleration at the affected occupant location, *j*. For determining the most severe likely vibration, engineering judgment is required to identify an unobstructed 5 to 10 ft long walking path as close as possible to the maximum mode shape value. The walking load location, *i*, should be placed near the midlength of the walking path because this location has the average mode shape value along the walking path. If the affected occupant location is unknown, then it should be located as closely as possible to the maximum mode shape value. For regular floor bays or pedestrian bridge spans, the walking load and affected occupant locations, *i* and *j*, respectively, can usually be conservatively located at midbay or midspan. The FRF minimum frequency should be approximately 1 Hz below the fundamental natural frequency and the maximum frequency should be approximately 20 Hz. The FRF magnitude should be computed at all natural frequencies plus 20 to 30 other frequencies between the minimum and maximum frequencies. The frequency of

Table 7-1. Harmonic Selection for High-Frequency Floors	
Dominant Frequency, Hz	***h***
9–11	5
11–13.2	6
13.2–15.4	7
15.4–17.6	8
17.6–20	9

maximum FRF magnitude is referred to as the dominant frequency.

The peak acceleration due to mode m is predicted using

$$a_{p,m} = 2\pi f_{n,m}\phi_{i,m}\phi_{j,m}I_{eff,m} \tag{7-4}$$

where

$f_{n,m}$ = natural frequency of mode m, Hz

$I_{eff,m}$ = effective impulse computed for mode m (Equation 1-6)

$\phi_{i,m}$ = mth mode mass-normalized shape value at the footstep

$\phi_{j,m}$ = mth mode mass-normalized shape value at the affected occupant

The effective impulse is a function of the step frequency, f_{step}, here taken as the dominant frequency divided by the harmonic number, h, from Table 7-1. The effective impulse also requires an estimate of bodyweight; Q = 168 lb is recommended.

The total response between the application of one footstep and the next is predicted using Equation 7-5, which is a superposition of the responses of all modes with frequencies not exceeding 20 Hz.

$$a(t) = \sum_{m=1}^{N_{Modes}} a_{p,m}e^{-2\pi f_{n,m}\beta t}\sin(2\pi f_{n,m}t) \tag{7-5}$$

The peak value of Equation 7-5 is not directly comparable to the sinusoidal peak acceleration tolerance limits in Figure 2-1. Therefore, the equivalent sinusoidal peak acceleration is computed using Equation 7-6, which is the product of the root-mean-square (RMS) of $a(t)$ between the application of two footsteps and the ratio of peak-to-RMS acceleration for a sinusoid, $\sqrt{2}$.

$$a_{ESPA} = \sqrt{2}\sqrt{\frac{1}{T}\int_0^T [a(t)]^2\,dt} \approx \sqrt{2}\sqrt{\frac{1}{N}\sum_{k=1}^{N} a_k^2} \tag{7-6}$$

where

N = number of discrete acceleration data points between one footstep and the next

T = footstep period, s

= $1/f_{step}$

a_k = kth acceleration data point

If the predicted equivalent sinusoidal peak acceleration does not exceed the applicable tolerance limit from Figure 2-1, the bay or span is predicted to be satisfactory.

Example 7.1—Office Floor with Cantilever Area

Given:

The floor system shown in Figure 7-5 is to be evaluated for vibration due to walking. The floor supports an electronic office space with typical mechanical and ceiling below. The slab thickness (total) is 5¼ in. with w_c = 110 pcf, f'_c = 3 ksi concrete on 2-in. deck weighing 2 psf. The spandrel supports cladding weighing 280 plf. Because the 16-ft-long cantilever between Gridlines E and F cannot be evaluated using the methods from Chapter 4, it is evaluated using finite element analysis. The bays between Gridlines D and E are also evaluated.

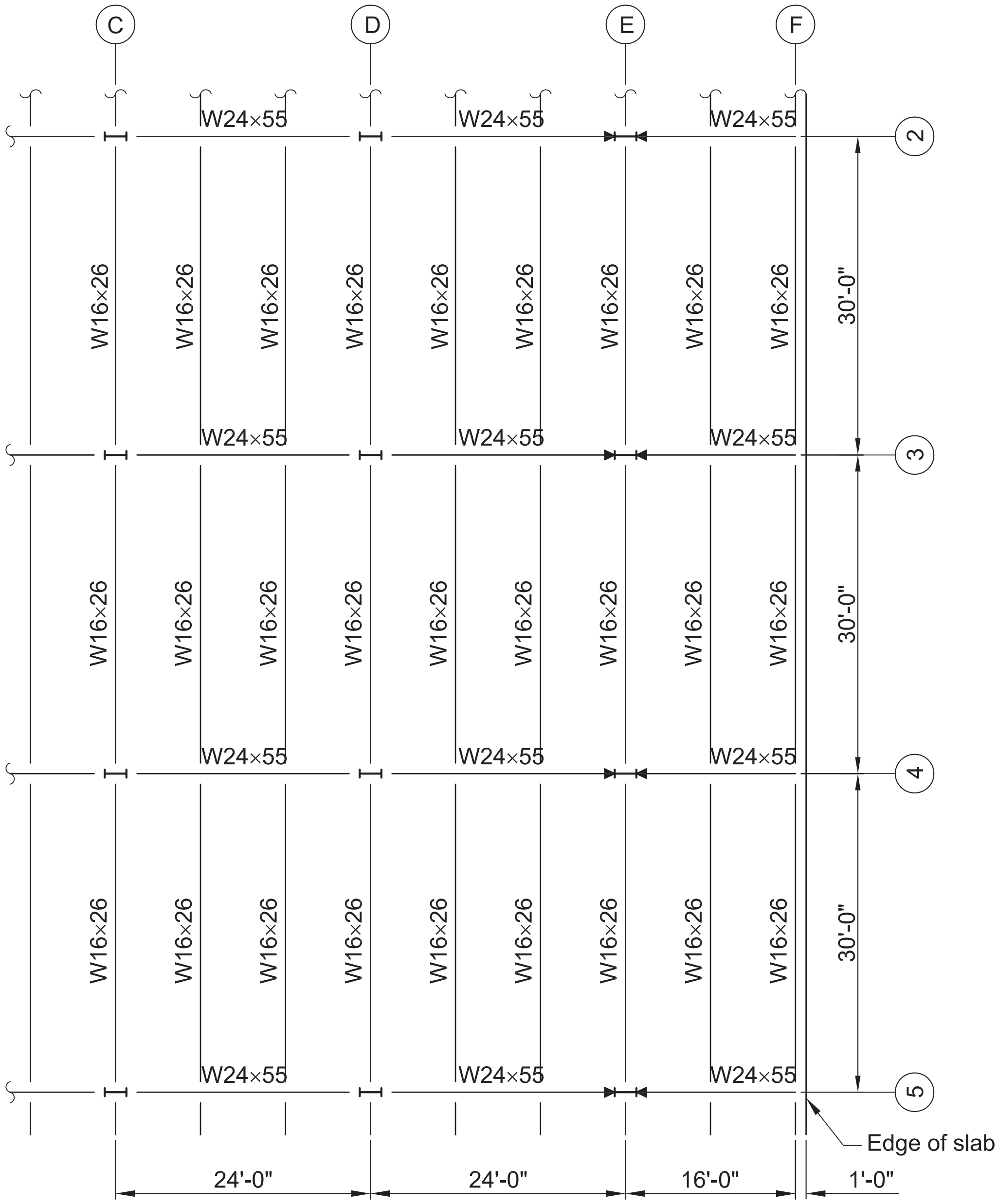


Fig. 7-5. Floor framing, Example 7.1.

Solution:

Masses

Uniform mass:

$$\text{Slab concrete} = (110\text{ pcf})\left(\frac{4.25\text{ in.}}{12\text{ in./ft}}\right)$$
$$= 39.0\text{ psf}$$

Deck = 2 psf

Typical mechanical, electrical, plumbing and ceiling = 4 psf (Section 3.3)

Electronic office build-out live load = 8 psf (Section 3.3)

$$\begin{aligned}\text{Total uniform load} &= 39.0\text{ psf} + 2.00\text{ psf} + 4.00\text{ psf} + 8.00\text{ psf}\\ &= 53.0\text{ psf}\end{aligned}$$

$$\begin{aligned}\text{Mass applied to shell elements} &= \frac{53.0\text{ psf}}{32.2\text{ ft/s}^2}\\ &= 1.65\text{ psf-s}^2\text{/ft}\end{aligned}$$

Cladding:

$$\begin{aligned}\text{Line mass} &= \frac{280\text{ plf}}{32.2\text{ ft/s}^2}\\ &= 8.70\text{ plf-s}^2\text{/ft}\end{aligned}$$

Member masses:

Computed by the program.

Viscous Damping Ratio

From Table 4-2:

$$\begin{aligned}\beta &= 0.01\text{ (structural system)} + 0.01\text{ (ceiling and ductwork)} + 0.005\text{ (electronic office fit-out)}\\ &= 0.025\end{aligned}$$

Slab Stiffness

$$\begin{aligned}1.35E_c &= 1.35w_c^{1.5}\sqrt{f_c'}\\ &= 1.35(110\text{ pcf})^{1.5}\sqrt{3\text{ ksi}}\\ &= 2{,}700\text{ ksi}\\ \nu &= 0.2\end{aligned}$$

Slab moments of inertia were computed using basic mechanics principles. For simplicity, only the concrete area and moment of inertia are included in the calculations.

Moment of inertia perpendicular to the ribs, considering only the concrete above the ribs = 34.3 in.^{4}/ft

Moment of inertia parallel to the ribs, considering concrete in and above the ribs = 102 in.^{4}/ft

Beam Transformed Moment of Inertia

For the typical W16×26 beams, with an effective concrete slab width of S = 8.00 ft < $0.4L_j$ = 0.4(30.0 ft) = 12.0 ft, considering only the concrete above the steel form deck, and using the dynamic elastic modulus of concrete, the transformed moment of inertia is 1,100 in.^{4} (See Example 4.1 for typical calculations.) For the W16×26 spandrel beams, the effective concrete slab width is 5 ft and the transformed moment of inertia is 1,010 in.^{4}, but it is increased 2.5 times to account for cladding restraint.

Girder Transformed Moment of Inertia

For the interior W24×55 girders, with an effective slab width of $0.4L_g = 0.4(24.0 \text{ ft}) = 9.60 \text{ ft} < L_j = 30.0 \text{ ft}$, and considering the concrete in the deck ribs, the transformed moment of inertia is 3,970 in.^{4} For the cantilevered girders, the effective slab width is $0.4L_g = 0.4(16.0 \text{ ft}) = 6.40 \text{ ft} < L_j = 30.0 \text{ ft}$, and the transformed moment of inertia is 3,670 in.^{4}

Model Development

The model in Figure 7-6 was created in a commercially available structural analysis program. Bay D-3/E-4 and the cantilevered area between E-3/F-4 will be evaluated. The model extends one bay to the west to Gridline C, and two bays to the north and two bays to the south, to Gridlines 1 and 6. Nodes are vertically restrained along Gridlines 1 and 6 to prevent motion at the boundaries of the model. For the same reason, beams along Gridline C are stiffened by a factor of 10 to prevent vertical movement along Gridline C. All members are modeled with continuous connections. Uniform mass is assigned to the shells and cladding line mass is assigned to each member along Gridline F.

Natural Mode Prediction

Fifteen modes are predicted between 3.49 Hz and 10 Hz. Selected mode shapes are shown in Figure 7-7.

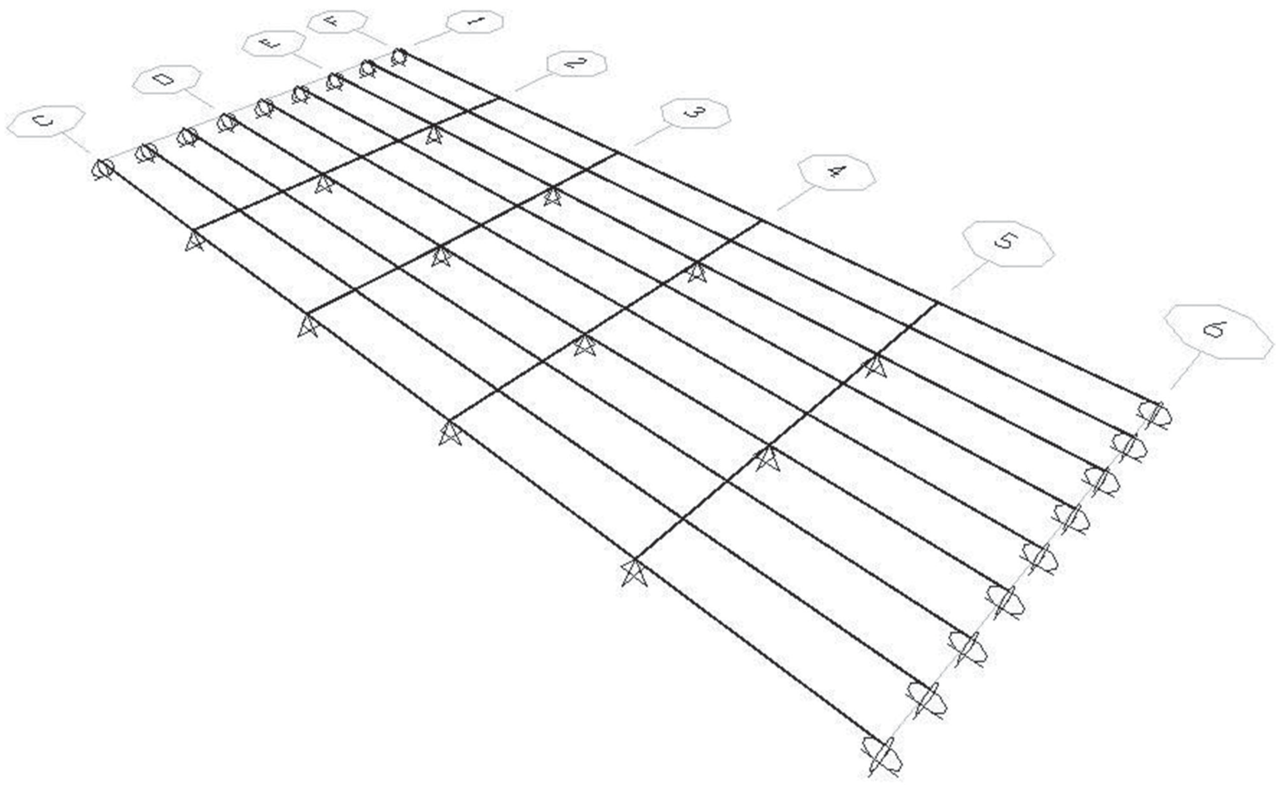


Fig. 7-6. Finite element model of floor framing, Example 7.1.

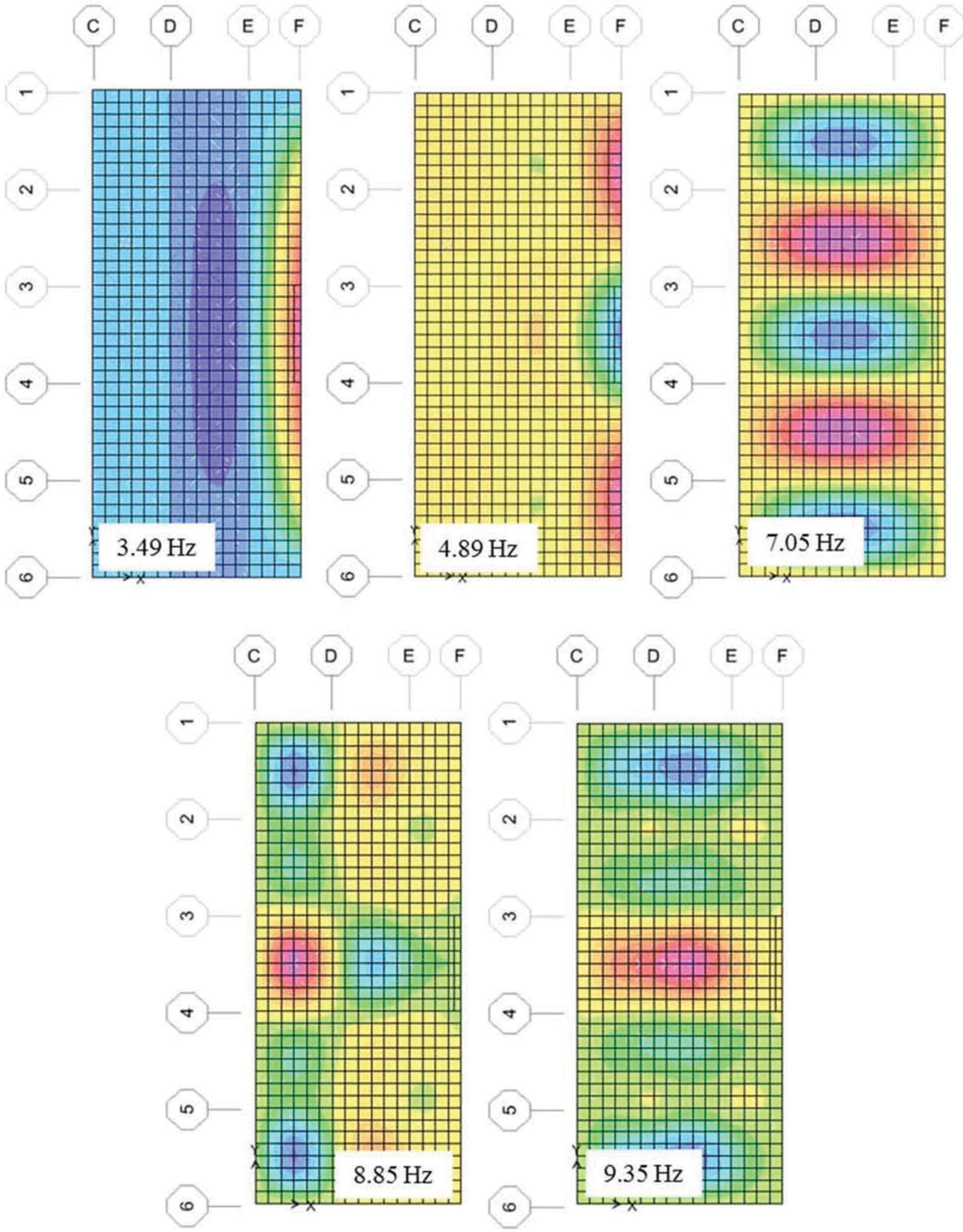


Fig. 7-7. Selected mode shapes, Example 7.1.

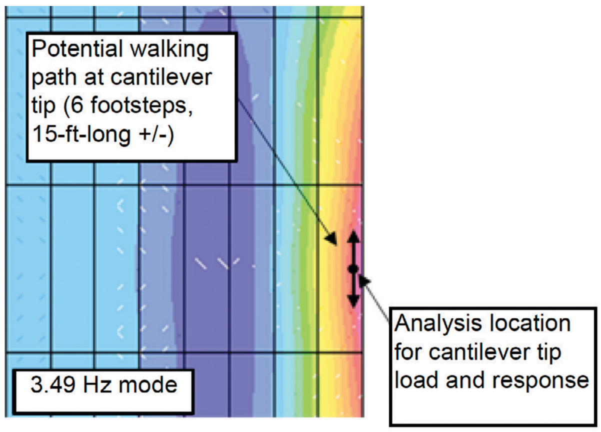
Frequency Response Function Prediction

Using engineering judgment and inspection of the mode shapes, analysis locations were located at the centers of potential walking paths. For example, Figure 7-8(a) shows the walking path and analysis location for the cantilever tip. Figure 7-8(b) shows both analysis locations.

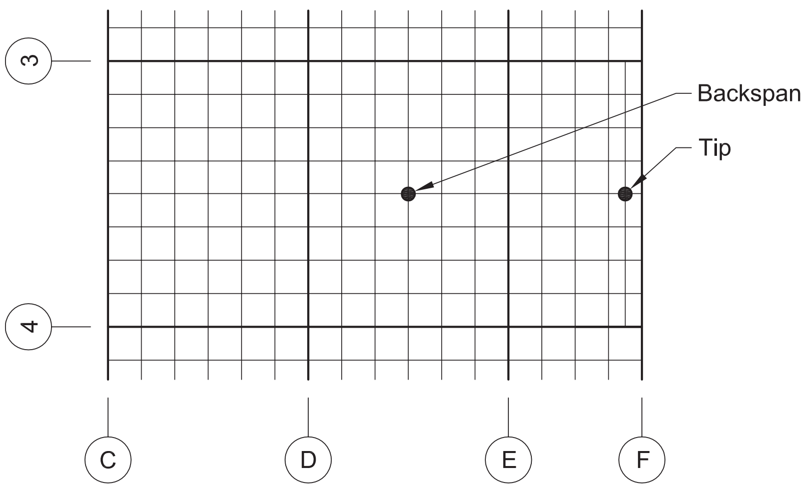
The frequency response functions shown in Figures 7-9 and 7-10 were computed for load and acceleration at the tip and backspan analysis locations. The cantilever tip FRF indicates a 3.49-Hz responsive mode, which is within reach of the second harmonic of walking, and a 4.89-Hz responsive mode, which is within reach of the third harmonic. The backspan FRF indicates 7.05-Hz and 8.85-Hz responsive modes within reach of the fourth harmonic of walking. Another responsive mode is at 9.35 Hz. The floor should not be vulnerable to vandal jumping (groups of people intentionally exciting the structure by jumping or bouncing in unison) or bouncing because no natural frequency is less than 3.0 Hz.

Predicted Maximum Acceleration Due to Walking at Cantilever

The cantilever (tip location) FRF magnitudes due to walking on the cantilever are shown in Figure 7-9.



(a) Cantilever tip



(b) Plan showing both analysis locations

Fig. 7-8. Backspan and tip analysis locations.

The peak acceleration due to excitation of the 3.49-Hz mode is computed as follows:

$$\alpha = 0.09e^{-0.075fn} \tag{7-2}$$
$$= 0.09e^{-0.075(3.49)}$$
$$= 0.069$$

For $\beta = 0.025$,

$$\rho = 12.5\beta + 0.625 \tag{7-3b}$$
$$= 12.5(0.025) + 0.625$$
$$= 0.938$$

From Figure 7-9, $FRF_{Max} = 0.0344\ \%g/\text{lb}$, and the predicted peak sinusoidal acceleration due to walking is

$$a_p = FRF_{Max}\alpha Q\rho \tag{7-1}$$
$$= (0.0344\ \%g/\text{lb})(0.069)(168\ \text{lb})(0.938)$$
$$= 0.374\%g < 0.5\%g \rightarrow \text{no complaints are predicted}$$

Similarly, the predicted peak acceleration due to excitation of the 4.89-Hz mode with $FRF_{Max} = 0.0362\ \%g/\text{lb}$, is $0.351\%g$, which does not exceed the $0.5\%g$ tolerance limit. Thus, no complaints are predicted.

Predicted Maximum Accelerations Due to Walking at Backspan

The backspan analysis station FRF magnitude shown in Figure 7-10 indicates responsive modes at 7.05 Hz, 7.95 Hz and 8.85 Hz. These are within reach of the fourth harmonic of the walking force; thus, the predicted maximum acceleration is determined using the low-frequency floor procedure in Section 7.4.1. The 9.35-Hz mode is more responsive than the other modes; its response is also evaluated. It is not within reach of the fourth harmonic of the walking force; therefore, the high-frequency procedure in Section 7.4.1 is used to predict the maximum acceleration for this mode.

Peak acceleration due to excitation of the 7.05 Hz mode is computed as follows:

$$\alpha = 0.09e^{-0.075f_n} \tag{7-2}$$
$$= 0.09e^{-0.075(7.05)}$$
$$= 0.0530$$

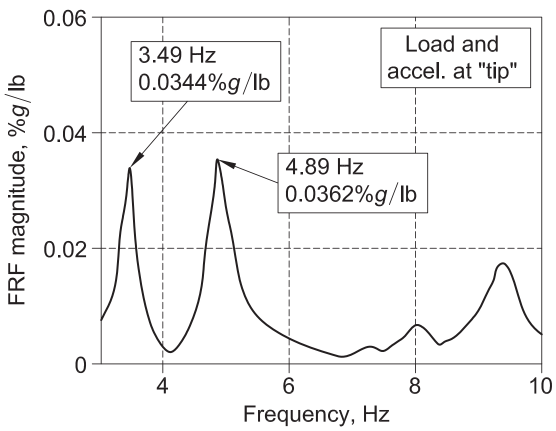


Fig. 7-9. Tip Location FRF magnitudes, Example 7.1.

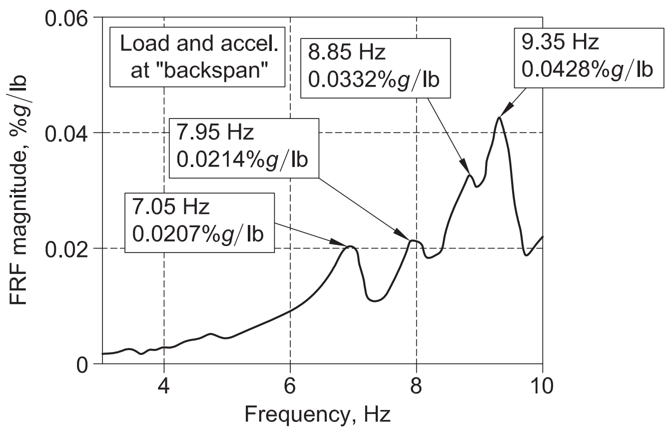


Fig. 7-10. Backspan location FRF magnitudes, Example 7.1.

$$\rho = 12.5\beta + 0.625 \tag{7-3b}$$
$$= 12.5(0.025) + 0.625$$
$$= 0.938$$

$$a_p = FRF_{Max}\alpha Q\rho \tag{7-1}$$
$$= (0.0207\ \%g/\text{lb})(0.0530)(168\ \text{lb})(0.938)$$
$$= 0.173\%g < 0.5\%g \rightarrow \text{no complaints are predicted.}$$

Similarly, the predicted peak acceleration due to excitation of the 8.85-Hz mode with $FRF_{Max} = 0.0332\%g/\text{lb}$, is $0.242\%g$, which does not exceed the tolerance limit in Figure 2-1 at 8.85 Hz, which is slightly greater than $0.5\%g$.

The FRF magnitudes at 7.95 Hz and 7.05 Hz are approximately equal, and α at 7.95 Hz is less than α at 7.05 Hz. Thus, the peak acceleration due to excitation of the 7.95-Hz mode is lower than $0.173\%g$.

No predicted acceleration exceeds $0.5\%g$; therefore, the floor is predicted to be satisfactory.

The high-frequency 9.35-Hz mode is evaluated for individual footstep impulse responses. The FRF magnitudes for all modes up to 20 Hz are found first. The 38 modes between 3.49 Hz and 20 Hz are summarized in Table 7-2. The 22nd mode at 12.6 Hz, shown in Figure 7-11, has the highest FRF magnitude; thus, it is used to find the controlling step frequency. From Table 7-1, the sixth harmonic can match the 12.6-Hz natural frequency; therefore, f_{step}, is 12.6 Hz/6 = 2.1 Hz. Because the program reports mass-normalized mode shapes, the units are $\left(\text{in./kip-s}^2\right)^{0.5}$.

The response of each mode is computed using Equations 7-4 and 7-5, where the mode shape values ϕ_i and ϕ_j are at the backspan node.

Using Mode 22 for example, the effective impulse is

$$I_{eff,22} = \left(\frac{f_{step}^{\ 1.43}}{f_{n,22}^{\ 1.30}}\right)\left(\frac{Q}{17.8}\right) \tag{1-6}$$
$$= \left(\frac{(2.1\ \text{Hz})^{1.43}}{(12.6\ \text{Hz})^{1.30}}\right)\left(\frac{168\ \text{lb}}{17.8}\right)$$
$$= 1.01\ \text{lb-s}$$

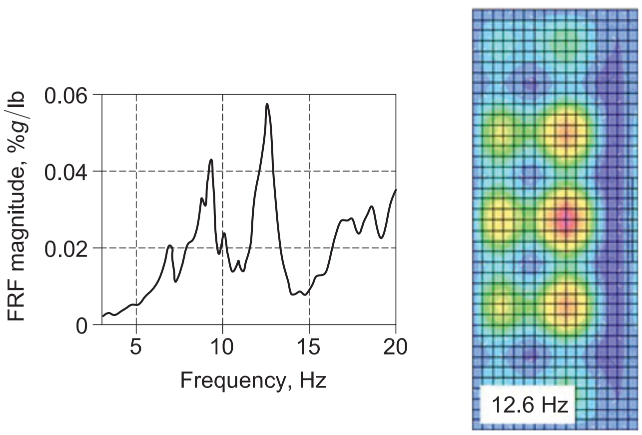


Fig. 7-11. FRF magnitude and Mode 22 shape, Example 7.1.

Table 7-2. Mass-Normalized Mode Shape Values, Example 7.1

Mode, *m*	$f_{n,m}$, Hz	$\phi_{Backspan}$*, (in./kip-s^2)$^{0.5}$	Mode, *m*	$f_{n,m}$, Hz	$\phi_{Backspan}$*, (in./kip-s^2)$^{0.5}$
1	3.49	0.449	20	12.0	1.93
2	3.80	0.000	21	12.6	0.000
3	4.89	–0.513	22	12.6	–3.15
4	6.88	0.000	23	13.1	0.000
5	7.05	1.67	24	13.3	0.286
6	7.31	–0.553	25	13.7	0.000
7	7.72	0.000	26	14.5	0.832
8	7.88	0.000	27	14.6	0.000
9	8.07	–1.47	28	14.7	0.044
10	8.85	1.83	29	15.5	1.00
11	8.98	0.000	30	16.0	0.000
12	9.35	–2.57	31	16.9	–1.47
13	9.42	0.565	32	17.3	0.000
14	9.63	0.000	33	17.5	1.18
15	10.1	–1.80	34	17.6	1.08
16	10.4	–0.649	35	17.7	0.000
17	10.7	0.000	36	18.6	1.93
18	10.9	–1.35	37	19.2	0.000
19	11.2	0.000	38	19.8	0.810

*At backspan location shown in Figure 7-8.

The peak acceleration due to Mode 22 from Equation 7-4, with $\phi_{i,22}$ from Table 7-2, is then

$$\begin{aligned} a_{p,22} &= 2\pi f_{n,22}\phi_{i,22}\phi_{j,22}I_{eff,22} \\ &= 2\pi(12.6 \text{ Hz})\left(-3.15\sqrt{\text{in./kip-s}^2}\right)^2\left(\frac{1.01 \text{ lb-s}}{1{,}000 \text{ lb/kip}}\right)\left(\frac{100\%g}{386 \text{ in./s}^2}\right) \\ &= 0.206\%g \end{aligned} \tag{7-4}$$

The acceleration waveform due to Mode 22 from Equation 7-5 is

$$\begin{aligned} a_{22}(t) &= a_{p,22}e^{-2\pi f_{n,22}\beta t}\sin(2\pi f_{n,22}t) \\ &= (0.206\%\text{g})e^{-2\pi(12.6 \text{ Hz})(0.025)t}\sin[2\pi(12.6 \text{ Hz})t] \end{aligned} \qquad \text{(from Eq. 7-5)}$$

All modal responses are computed and superimposed over the footstep period, $T_{step} = 1/f_{step} = 0.476$ s, to obtain the total response. The Mode 22 and total responses are shown in Figure 7-12.

Because the peak acceleration, 0.865%g, is not comparable to sinusoidal acceleration tolerance limits, the equivalent sinusoidal peak acceleration (ESPA) is computed using Equation 7-6. The total acceleration equation is sampled at 0.005 s to generate a vector of N discrete accelerations, a_k, over the footstep period. The ESPA is

$$\begin{aligned} a_{ESPA} &= \sqrt{\frac{2}{N}\sum_{k=1}^{N}a_k^2} \\ &= 0.314\%g \end{aligned} \qquad \text{(from Eq. 7-6)}$$

The tolerance limit conservatively selected from Figure 2-1 for 9.35-Hz vibration is just over 0.5%g. The predicted acceleration, 0.314%g, is less than the tolerance limit; therefore, no complaints are expected.

Because the predicted accelerations for both the cantilever and backspan areas are below the tolerance limit, the floor is predicted to be satisfactory.

7.4.2 Running on Level Floors and Tracks

This section provides an evaluation method for level surfaces such as floors and tracks subject to running. Because the running step frequency range is wide (Table 1-1), it is usually possible for a force harmonic frequency to match a responsive natural frequency and cause a resonant build-up. The resonant response can be predicted using the FRF method described in Section 7.4.1 with several modifications. The evaluation is unsatisfactory if the predicted sinusoidal peak acceleration exceeds the applicable human comfort tolerance limit in Section 2.1.

The FRF magnitude is computed for unit load at the running load location, and acceleration is computed at the affected occupant location. Surfaces subject to running usually have few obstructions, and nonparticipating occupants can usually be anywhere; therefore, the running load and affected occupant locations should be conservatively located at the maximum mode shape amplitude. The FRF minimum frequency should be approximately 1 Hz below the fundamental natural frequency, and the maximum frequency should be at least 17 Hz, which is 1 Hz above the maximum fourth harmonic forcing frequency associated with running. The FRF magnitude must be computed at all natural frequencies, plus 20 to 30 other frequencies between the minimum and maximum frequencies. The frequency below 17 Hz, with maximum FRF magnitude, is referred to as the dominant frequency. If the dominant frequency is below 3 Hz, the floor or pedestrian bridge will be vulnerable to vandal jumping (groups of people intentionally exciting the structure by jumping or bouncing in unison); thus, the structure should also be evaluated for group rhythmic loads per Section 7.4.4.

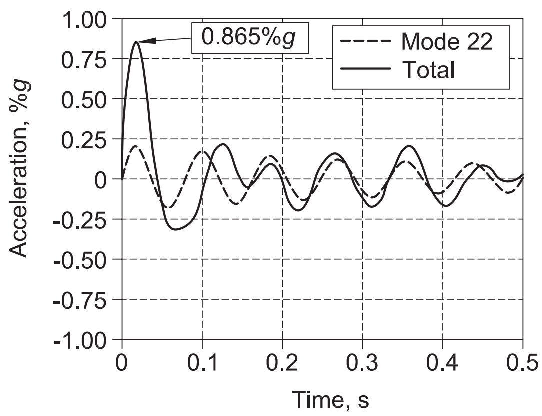


Fig. 7-12. Mode 22 and total responses for Example 7.1.

Human running force is represented by the Fourier series in Section 1.6, with step frequency, f_{step}, equal to the dominant frequency divided by the harmonic number, h, from Table 7-3. The dynamic coefficient, α, is also listed in Table 7-3. For typical applications, a bodyweight, Q, of 168 lb is recommended. For other groups, e.g., football players, average or maximum bodyweight should be considered.

The peak acceleration due to running is

$$a_p = FRF_{Max}\alpha_h Q\left[1 - e^{-2\pi\beta h N_{Steps}}\right] \tag{7-7}$$

where

FRF_{Max} = maximum FRF magnitude, %g/lb
N_{Steps} = number of footsteps required to cross the bay or span ≤ 10
Q = bodyweight
h = number of the harmonic that causes resonance (Table 7-3)
α_h = dynamic coefficient (Table 7-3)
β = viscous damping ratio (Section 4.3)

If runner synchronization is likely, as may be the case near the beginning of a sprinting race, the response due to an individual in Equation 7-7 should be amplified. Based on research by Pernica (1990) and Bachmann and Amman (1987), the acceleration due to a group of runners is the product of the acceleration due to an individual and the minimum of 2.0 or $\sqrt{n}$ where n is the anticipated number of runners.

If the predicted peak acceleration does not exceed the applicable tolerance limit discussed in Chapter 2 from Figure 2-1, the bay or span is predicted to be satisfactory.

7.4.3 Walking and Running on Slender Stairs

This section provides an evaluation method based on research by Davis and Murray (2009) and Davis and Avci (2015) for slender monumental stairs (not typical pan-type or other short and stiff stairs) subject to individual or group descents, which are always more severe than ascents. Because the stair descent step frequency range is wide (Table 1-1), it is usually possible for a force harmonic frequency to match a responsive natural frequency and cause a resonant build-up. The resonant response can be predicted using the FRF method described in Section 7.4.1 with several modifications. The structure is satisfactory if predicted sinusoidal peak acceleration does not exceed the applicable human comfort tolerance limit recommended in Table 4-5.

Table 7-3. Fourier Series Parameter Selection for Running		
Dominant Frequency, Hz	***h***	**α_h**
1.6–4	1	1.4
4–8	2	0.4
8–12	3	0.2
12–16	4	0.1

The FRF magnitude is computed for vertical unit load at the walking load location and vertical acceleration at the affected occupant location. The walker load location must be identified using engineering judgment. Resonant build-up durations are highly variable, in the range of five to 10 steps long; therefore, it is recommended that a seven- or eight-step resonant build-up be used for design. The seven or eight consecutive steps are to be located as close as possible to the maximum mode shape value and are used to select the walking load location at a node near the middle of the selected steps. The affected occupant location must also be identified using engineering judgment, noting that occupants must be stationary to feel the vibrations. Thus, the affected occupant location is usually at an intermediate landing, if one exists, and at midspan, otherwise. The FRF minimum frequency should be at least 1 Hz below the fundamental natural frequency, and the maximum frequency should be approximately 10 to 12 Hz. The FRF magnitude is then computed at all natural frequencies, plus 20 to 30 other frequencies between the minimum and maximum frequencies. The maximum FRF magnitude is then identified, and its frequency is referred to as the dominant frequency. If the dominant frequency is below 4.5 to 5.0 Hz, the first harmonic of the walking force might match the dominant frequency and cause resonance and very high responses. Therefore, it is recommended that stairs be designed with dominant frequencies not less than 5 Hz. If the dominant frequency is below 3 Hz, the stair will also be vulnerable to vandal jumping, and should also be evaluated for group rhythmic loads per Section 7.4.4.

The presence of stationary occupants on a stair can affect the descent footstep frequency. Typically, walkers descend stairs with step frequencies below approximately 2.5 Hz in the presence of stationary occupants; this regular descent load case should always be considered. If the stair is wide enough, occupants might be able to descend rapidly, at frequencies between 2.5 Hz and 4.0 Hz, in the presence of stationary occupants. Similarly, if the stair is wide enough, a group of occupants might be able to descend rapidly in the presence of stationary occupants. Engineering judgment and communication with the architect or owner should be used to decide if a rapidly descending individual or group is a realistic load case. The Section 4.3 subsection "Recommended Evaluation Procedures" and Example 4.6 provide additional information.

The peak acceleration due to an individual descending the stair is

$$a_p = 0.62e^{-\gamma f_n} FRF_{Max} RQ(1 - e^{-100\beta}) \tag{7-8}$$

where

FRF_{Max} = maximum FRF magnitude, $\%g$/lb

Q = bodyweight = 168 lb

β = viscous damping ratio (Section 4.3)

γ = 0.29 for normal descents; 0.19 for rapid descents

The calibration factor, R, is 0.7 for normal descents. For rapid descents, $R = 0.5$ if $f_n \leq 8$ Hz, or 0.7 otherwise.

The acceleration caused by a rapidly descending group is triple the acceleration due to a rapidly descending individual.

7.4.4 Rhythmic Activity on Floors and Balconies

This section provides an evaluation method for floors and balconies subject to rhythmic group loads such as dancing or aerobics. The resonant response is predicted using the FRF method, in which the predicted peak sinusoidal acceleration is the product of FRF magnitude and force harmonic amplitude. The evaluation is satisfactory if predicted sinusoidal peak acceleration does not exceed the applicable human comfort tolerance limit from Figure 2-1 or Table 5-1.

It is recommended that the FRF magnitude be computed for unit uniform load covering the anticipated group load area and vertical acceleration be determined at the affected occupant location. The FRF minimum frequency should be 1 Hz and the maximum frequency 12 Hz, which exceeds the forcing function maximum harmonic frequency. The FRF magnitude should be computed at all natural frequencies, plus 20 to 30 other frequencies between the minimum and maximum frequencies. The frequency at the maximum FRF magnitude is the dominant frequency.

Dynamic forces during rhythmic group loads are represented by the Equation 1-8 Fourier series, with the step frequency selected such that its minimum possible harmonic has a frequency matching the dominant frequency. The Fourier series parameters required for the FRF method are given in Table 7-4. Participant weight, w_p, is an estimate for each

Table 7-4. Fourier Series Parameters for Rhythmic Group Loads		
Group Dancing, w_p = 12.5 psf		
Dominant Frequency, Hz	h	α
1.5–2.5	1	0.50
2.5–5	2	0.05
Lively Concert or Sports Event, w_p = 31 psf		
Dominant Frequency, Hz	h	α
1.5–3	1	0.25
3–6	2	0.05
Aerobics, w_p = 4.2 psf		
Dominant Frequency, Hz	h	α
2.0–2.75	1	1.5
2.75–5.5	2	0.6
5.5–8.25	3	0.1
Jumping Exercises, w_p = 4.2 psf		
Dominant Frequency, Hz	h	α
2.0–2.75	1	1.8
2.75–5.5	2	1.3
5.5–8.25	3	0.7
8.25–11	4	0.2

activity and should be adjusted if the anticipated participant weight is significantly different from the listed value. The harmonic frequencies are determined by (1) selecting the harmonic, h, to match the dominant frequency; (2) computing the step frequency, f_{step} (taken as the dominant frequency divided by h); and (3) computing each harmonic frequency, if_{step}, where i is the harmonic number. The harmonic force amplitudes are $w_p\alpha_i$.

If the dominant frequency exceeds the maximum harmonic frequency, f_{step} is selected such that the predicted response is maximized. This is illustrated in Example 7.2.

The predicted peak acceleration due to each force harmonic is the product of the FRF magnitude, %g/psf, at the harmonic frequency, if_{step}, and the harmonic load amplitude:

$$a_{p,i} = FRF(if_{step})\alpha_i w_p \tag{7-9}$$

To predict the peak acceleration, the peak accelerations due to all force harmonics are combined using the 1.5 power rule:

$$a_p = \left[\sum_i \left(a_{p,i}\right)^{1.5}\right]^{1/1.5} \tag{7-10}$$

Example 7.2—Balcony Subject to Rhythmic Loads

Given:

The balcony framing shown in Figure 7-13 is to be evaluated for vibration due to a lively concert. The slab is 3½ in. thick (total), with w_c = 145 pcf, f'_c = 3.5 ksi concrete on 9⁄16-in. deck weighing 1 psf. The short riser walls at each step are 3.5 in. thick. Seats are at approximately 3-ft spacing in each direction. The front edge of the balcony supports a guardrail weighing 30 plf. Because its geometry falls outside the scope of floor framing covered by Chapter 5, it is evaluated using FEA. The beams and girders are ASTM A992 material.

Masses

Uniform slab mass:

$$\text{Slab concrete} = (145 \text{ pcf})\left(\frac{3.22 \text{ in.}}{12 \text{ in./ft}}\right)$$
$$= 38.9 \text{ psf}$$

$$\text{Deck} = 1 \text{ psf}$$

$$\text{Occupants} = \frac{168 \text{ lb}}{(3 \text{ ft})(3 \text{ ft})}$$
$$= 18.7 \text{ psf}$$

$$\text{Seats (assumed)} = \frac{45.0 \text{ lb}}{(3.00 \text{ ft})(3.00 \text{ ft})}$$
$$= 5.00 \text{ psf}$$

$$\text{Ceiling} = 2 \text{ psf}$$

$$\text{Total deck + Superimposed loads} = 38.9 \text{ psf} + 1.00 \text{ psf} + 18.7 \text{ psf} + 5.00 \text{ psf} + 2.00 \text{ psf}$$
$$= 65.6 \text{ psf}$$

$$\text{Total deck + Superimposed loads to the shells} = \frac{65.6 \text{ psf}}{32.2 \text{ ft/s}^2}$$
$$= 2.04 \text{ psf-s}^2\text{/ft}$$

Riser walls:

Computed by the program.

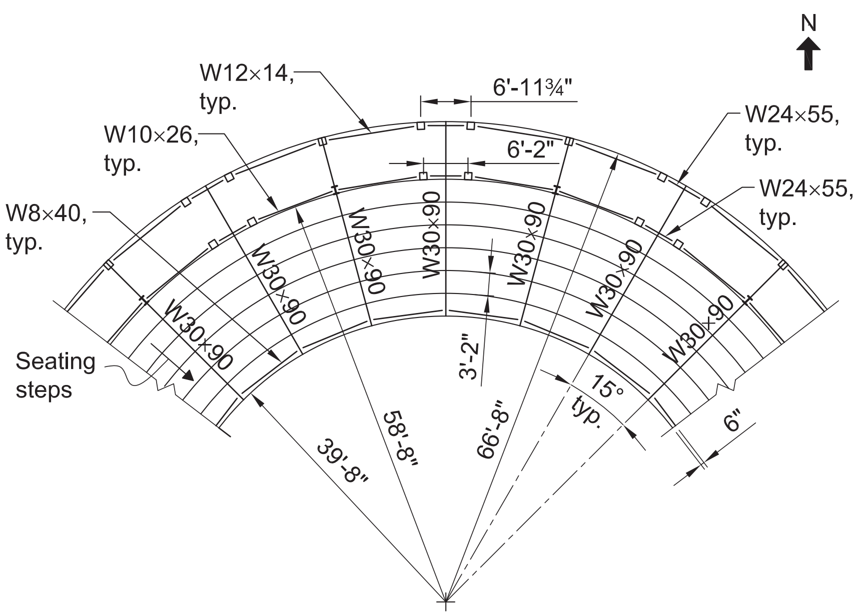


Fig. 7-13. Framing plan, Example 7.2.

Guardrail:

$$\text{Line mass} = \frac{30.0\text{ plf}}{32.2\text{ ft/s}^2}$$
$$= 0.932\text{ plf-s}^2\text{/ft}$$

Member masses:

Computed by the program.

Damping

6% of critical viscous damping due to crowd presence on the balcony as recommended in Chapter 5.

Slab Stiffnesses

$1.35E_c = 4{,}410$ ksi
Moment of inertia parallel to the deck = 34.9 in.^{4}/ft
Moment of inertia perpendicular to the deck = 25.3 in.^{4}/ft

Member Stiffnesses

Noncomposite member stiffnesses are used in the model because there are physical separations between the steel members and the underside of the steel deck.

Model Development

A finite element model was created in a commercially available structural analysis program. Slabs and walls were modeled with orthotropic and isotropic shell elements, respectively. Frame elements were modeled using regular frame elements. Shells were connected to frame elements using massless rigid links as shown in Figure 7-14. Due to its circular arc (radial) geometry, tangential membrane stresses significantly add to the vertical stiffness. However, minuscule movements—present in reality, but not present in the model—will probably relieve such stresses. Thus, tangential membrane stiffnesses were decreased to nearly zero along gridlines selected using engineering judgment. Uniform mass is assigned to the shells and cladding line mass is assigned along the front of the balcony.

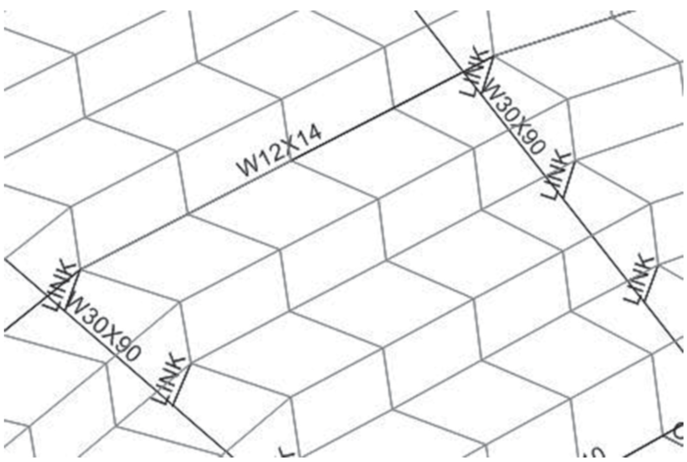


Fig. 7-14. Massless rigid links connecting frame members and shells, Example 7.2.

Natural Mode Prediction

The two modes shown in Figure 7-15 were predicted using standard eigenvalue analysis.

Frequency Response Function Prediction

Frequency response functions were computed for two potentially critical cases. In Load Case 1, a group applies rhythmic excitation to the left or right side of the balcony in the areas in motion for the 7.21-Hz mode. The left and right sides synchronously move in opposite directions; therefore, applying load on the left and right side simultaneously is unconservative. In Load Case 2, a group applies rhythmic excitation to the portions of the balcony in motion for the 8.25-Hz mode. Approximate load extents are shown in Figure 7-16. In each case, acceleration was computed at the cantilever tip near the center of the loaded area. The computed FRF magnitudes, in %g/psf units, are in Figure 7-17. Each FRF has a maximum at 8.40 Hz, even though the nearest natural frequency is 8.25 Hz. The FRF magnitude is the summation of FRFs contributed by each mode, and occasionally the maximum magnitude is between natural frequencies, as is the case here.

Predicted Acceleration Due to Lively Concert

The estimated weight of participants, w_p, is 18.7 psf. From Table 7-4, the dynamic coefficients for lively concerts are $\alpha_1 = 0.25$ and $\alpha_2 = 0.05$, and the excitation frequency, f_{step}, is between 1.5 Hz and 3 Hz. The maximum frequency of the second harmonic is 6 Hz, which is less than the fundamental frequency, 7.21 Hz, so it is not possible for a force harmonic to match a natural frequency and cause resonance. The FRF magnitude indicates the maximum response will occur when the step frequency is at its maximum value, 3 Hz; hence, $f_{step} = 3$ Hz. The Load Case 1 FRF magnitudes at 3 Hz and 6 Hz are higher than the corresponding Load Case 2 FRF magnitudes; thus, Load Case 1 is used to evaluate the balcony.

The peak acceleration due to the first harmonic, using Equation 7-9, is

$$\begin{aligned} a_{p,1} &= \text{FRF}(f_{step})\alpha_1 w_p \qquad \text{(from Eq. 7-9)} \\ &= (0.238\%g/\text{psf})(0.25)(18.7\text{ psf}) \\ &= 1.11\%g \end{aligned}$$

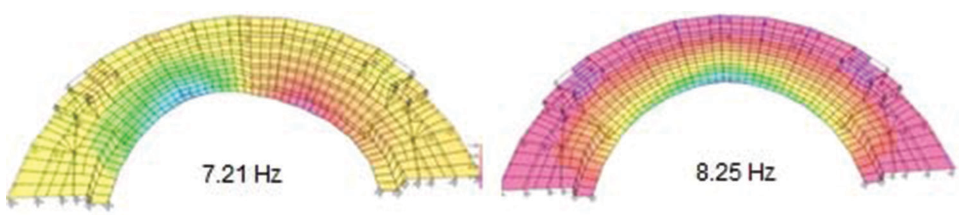


Fig. 7-15. Predicted natural modes, Example 7.2.

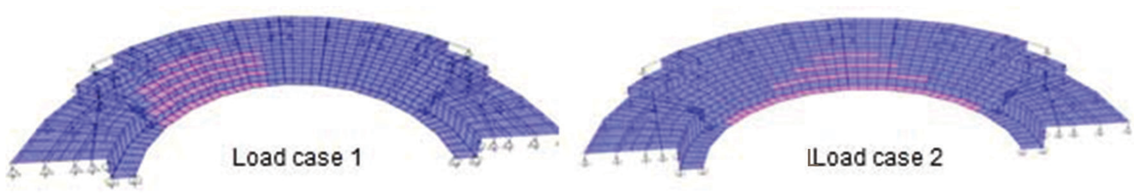


Fig. 7-16. Dynamic load locations, Example 7.2.

The peak acceleration due to the second harmonic is

$$a_{p,2} = \text{FRF}(2f_{step})\alpha_2 w_p \qquad \text{(from Eq. 7-9)}$$
$$= (2.01\%g/\text{psf})(0.05)(18.7 \text{ psf})$$
$$= 1.88\%g$$

The total peak acceleration, computed using the 1.5 power rule, Equation 7-10, is

$$a_p = \left[\sum_i \left(a_{p,i}\right)^{1.5}\right]^{1/1.5} \qquad (7\text{-}10)$$
$$= \left[(1.11)^{1.5} + (1.88)^{1.5}\right]^{1/1.5}$$
$$= 2.41\%g$$

From Table 5-1, participants in the rhythmic activity will likely tolerate accelerations between 4%g and 7%g; therefore, no complaints are expected, and the balcony framing is satisfactory.

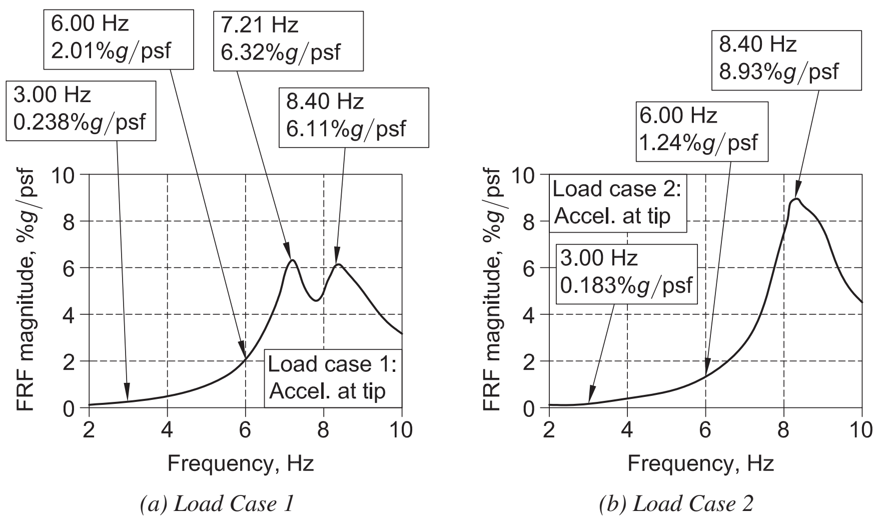


Fig. 7-17. Predicted FRF magnitudes, Example 7.2.

7.5 EVALUATION OF FLOORS SUPPORTING SENSITIVE EQUIPMENT

This section provides evaluation methods for floors supporting vibration-sensitive equipment. As explained in Chapter 6, equipment manufacturers often provide specific tolerance limits, usually expressed as waveform peak acceleration or narrowband spectral acceleration magnitude. Generic tolerance limits, expressed as one-third octave spectral velocity magnitudes, are available for many classes of equipment, and should be used when specific limits are not available. The following sections provide FEA methods for response prediction in measures that match common tolerance limit forms. Each response prediction equation includes a calibration factor which resulted in a 10% probability that measured response exceeded predicted response in a large database of measurements (Liu, 2015; Liu and Davis, 2015).

7.5.1 Conceptual Models of Floor Vibrations Due to Footfalls

Floor bays with dominant frequencies lower than the fourth harmonic maximum frequency undergo resonant responses similar to the one shown in Figure 1-8(a). In contrast, floors with higher dominant frequencies undergo impulse responses such as the one shown in Figure 1-4(a). Floors with dominant frequencies approximately equal to the fourth harmonic maximum frequency probably exhibit intermediate behavior between resonant and impulse responses. In this section, walking is characterized as very slow, slow, moderate or fast. See Section 6.1.3 for a more detailed discussion.

For very slow walking, waveform peak acceleration is computed using Section 7.5.3. For slow, moderate and fast walking, waveform peak acceleration is computed using Section 7.5.2 if the dominant natural frequency does not exceed the fourth harmonic maximum frequency, f_{4max}, from Table 6-1, or otherwise using Section 7.5.3.

For very slow walking, spectral responses are computed using Section 7.5.3. For slow, moderate and fast walking, spectral responses are computed using Section 7.5.2 if the dominant natural frequency does not exceed the intermediate zone lower boundary, f_L, from Table 6-1, or using Section 7.5.3 if the dominant frequency exceeds f_U. If the dominant frequency is between f_L and f_U, the response is predicted by linear interpolation between the resonant response at f_L and the impulse response at f_U.

7.5.2 Resonant Response

The resonant response can be predicted using the FRF method in which the predicted peak acceleration is the product of the maximum FRF magnitude and force harmonic amplitude.

The FRF magnitude is computed for vertical unit load at the walking load location and vertical acceleration at the equipment location. An average resonant build-up corresponds to approximately six footsteps with an average stride length of approximately 2.5 ft, which means the average resonant build-up is due to footsteps along a path approximately 15 ft long. Engineering judgment is needed to identify an unobstructed walking path as close as possible to the maximum mode shape value. The walking load location should be set near the midlength of the walking path. For resonant response predictions, the frequency below f_{4max} (Table 6-1) with a maximum FRF magnitude is referred to as the dominant frequency. Note this is not always a natural frequency due to contributions from multiple modes, especially when damping is high and there are numerous closely-spaced modes.

Predicting Waveform Peak Acceleration

The waveform peak acceleration is

$$a_p = 1.3 FRF_{Max} \alpha Q \tag{7-11}$$

where

FRF_{Max} = maximum FRF magnitude at frequencies below f_{4max} (Table 6-1), $\%g$/lb

Q = bodyweight, 168 lb

The dynamic coefficient is

$$\alpha = 0.1 e^{-\gamma f_n} \tag{7-12}$$

where $\gamma = 0.1$ for slow walking, 0.09 for moderate speed walking, or 0.08 for fast walking.

Predicting Narrowband Spectral Acceleration

The narrowband spectral acceleration maximum RMS magnitude is approximated by

$$A_{NB} = a_p \frac{4.3}{T f_{step}} \tag{7-13}$$

where

T = walking event duration, taken as 8 s

a_p = peak acceleration from Equation 7-11

f_{step} = 1.6 Hz for slow walking, 1.85 Hz for moderate speed walking, or 2.1 Hz for fast walking

Alternatively, the narrowband spectral acceleration can be computed by (1) constructing the walking event waveform, defined by Equations 7-14a (build-up) and 7-14b (decay) at 0.01 sec. intervals; (2) sampling the resulting waveform at 0.01 s intervals; and (3) computing the fast Fourier

Table 7-5. One-Third Octave Bands

Band No.	Frequencies, Hz		
	Lower Limit, f_1	Center, f_{ctr}	Upper Limit, f_2
6	3.55	4	4.47
7	4.47	5	5.62
8	5.62	6.3	7.08
9	7.08	8	8.91
10	8.91	10	11.2
11	11.2	12.5	14.1
12	14.1	16	17.8
13	17.8	20	22.4

transformation, which will provide the magnitude at each frequency

$$a(t) = a_p \frac{\cos(2\pi f_n t)\left(e^{-\beta 2\pi f_n t} - 1\right)}{\left|\cos\left(2\pi f_n T_{BU}\right)\left(e^{-\beta 2\pi f_n T_{BU}} - 1\right)\right|} \text{ if } t \le T_{BU} \tag{7-14a}$$

$$a(t) = -a_p \cos\left[2\pi f_n (t - T_{BU})\right] e^{-\beta 2\pi f_n (t - T_{BU})} \text{ if t} > T_{BU} \tag{7-14b}$$

where resonant build-up duration, T_{BU}, is computed using Equation 7-15, and with N_{step} approximately equal to 6, adjusted slightly if necessary, to obtain continuity of Equations 7-14a and 7-14b at T_{BU}:

$$T_{BU} = \frac{N_{step}}{f_{step}} \tag{7-15}$$

Predicting One-Third Octave Velocity Spectrum

The one-third octave velocity maximum RMS magnitude is approximated by

$$V_{1/3} = 0.8 \frac{A_{NB}}{2\pi} \sqrt{\frac{T}{30 f_n}} \tag{7-16}$$

where A_{NB} and T are defined previously and f_n is the dominant frequency.

Alternatively, the one-third octave velocity spectrum can be approximated by converting the narrowband acceleration spectrum to spectral velocities in the standard one-third octave bands shown in Table 7-5 as follows. First, the narrowband acceleration spectrum is converted to a narrowband velocity spectrum by dividing the acceleration at each narrowband frequency, $A_{NB}(f)$, by $2\pi f$. Second, the energy spectral density, *ESD*, attributed to each narrowband frequency is computed using Equation 7-17.

$$ESD(f) = \frac{\left[V(f)\right]^2}{\Delta f} \tag{7-17}$$

where

$V(f)$ = narrowband spectral velocity at frequency, f, mips

Δf = narrowband frequency resolution, 0.125 Hz, if T = 8 s

The energy attributed to each one-third octave band is the sum of the energies attributed to the narrow bands within the one-third octave band:

$$E_{1/3} = \int_{f_1}^{f_2} ESD(f)\, df \tag{7-18}$$

where

f_1 and f_2 = one-third octave band lower and upper limits from Table 7-5

Finally, the one-third octave spectral velocity in each one-third octave band is the velocity amplitude (RMS) with energy equivalent to the total energy in the one-third octave band:

$$V(f_{ctr}) = 0.8\sqrt{E_{1/3}} \tag{7-19}$$

where

$V(f_{ctr})$ = spectral velocity at one-third octave band centered at f_{ctr}, mips

7.5.3 Impulse Response

Predicting Peak Acceleration

The peak acceleration due to an individual footstep can be computed using the effective impulse method described in Section 1.5, based on the research by Willford et al. (2006, 2007) and Liu and Davis (2015).

The FRF magnitude is computed for unit load at the walking load location, i, and acceleration at the equipment location, j. Engineering judgment is needed to identify an unobstructed 5- to 10-ft-long walking path as close as possible to the maximum mode shape value. The walking load location is placed near the midlength of the walking path. The FRF minimum frequency should be approximately 1 Hz below the fundamental natural frequency, and the maximum frequency should be approximately 20 Hz. The dominant frequency is taken as the frequency which causes the maximum FRF magnitude.

The predicted peak acceleration due to mode m is

$$a_{p,m} = 2\pi f_{n,m}\phi_{i,m}\phi_{j,m}I_{eff,m} \tag{7-20}$$

where

$I_{eff,m}$ = effective impulse computed for mode m (Equation 1-6)

$f_{n,m}$ = natural frequency of mode m, Hz

$\phi_{i,m}$ = mth mode mass-normalized shape value at the footstep

$\phi_{j,m}$ = mth mode mass-normalized shape value at the equipment

The effective impulse calculation requires the step frequency, f_{step}, which is the dominant frequency divided by h, from Tables 7-6 through 7-9. The variable, h, is the minimum harmonic with frequency that can match the dominant frequency. The effective impulse also requires an estimate of bodyweight; Q = 168 lb is recommended.

The total response between the application of one footstep and the application of the next is

$$a(t) = \sum_{m=1}^{N_{Modes}} a_{p,m}e^{-2\pi f_{n,m}\beta t}\sin(2\pi f_{n,m}t) \tag{7-21}$$

The calibrated peak acceleration, a_p, is

$$a_p = 1.5\max\left[\left|a(t)\right|\right] \tag{7-22}$$

where 1.5 is the calibration factor.

Predicting Narrowband Spectral Acceleration

The spectral acceleration maximum magnitude can be approximated by

$$A_{NB} = a_p\,\frac{1-e^{-2\pi\beta h}}{20\beta h} \tag{7-23}$$

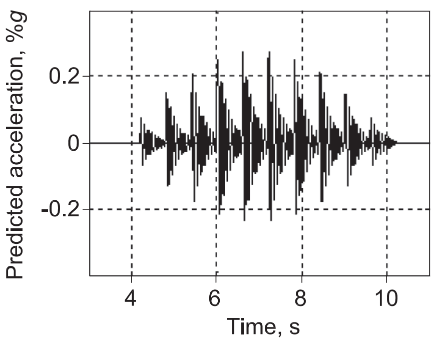
Alternatively, the narrowband acceleration magnitude can be found by (1) computing responses to individual footsteps at different locations along the walking path using Equation 7-21; (2) appending individual footstep responses to generate the waveform due to the entire walking event, as shown in Figure 7-18; (3) fast Fourier transforming the waveform to the corresponding narrowband spectrum; and (4) multiplying the magnitude by the 1.5 calibration factor.

Predicting One-Third Octave Velocity Magnitude

The one-third octave velocity maximum RMS magnitude is approximated by

$$V_{1/3} = 0.8\,\frac{A_{NB}}{2\pi}\sqrt{\frac{T}{30 f_n}} \tag{7-24}$$

Alternatively, the one-third octave velocity magnitude can be computed using the bandwidth conversion procedure described in Section 7.5.2 for resonant responses.



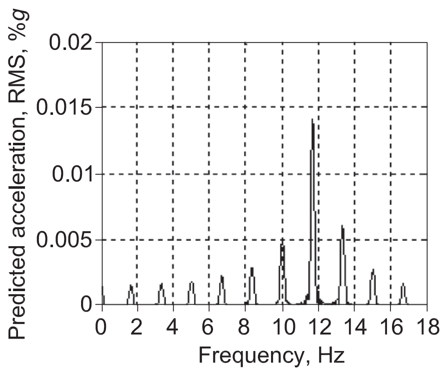


Fig. 7-18. Example of constructed waveform and corresponding spectrum.

Table 7-6. Harmonic Selection for Very Slow Walking (1.0–1.5 Hz)	
Dominant Frequency, Hz	***h***
4.0–6.0	4
6.0–7.5	5
7.5–9.0	6
9.0–10.5	7
10.5–12.0	8
12.0–13.5	9
13.5–15.0	10
15.0–16.5	11
16.5–18.0	12
18.0–19.5	13

Table 7-7. Harmonic Selection for Slow Walking (1.5–1.7 Hz)	
Dominant Frequency, Hz	***h***
6.8–8.5	5
8.5–10.2	6
10.2–11.9	7
11.9–13.6	8
13.6–15.3	9
15.3–17.0	10
17.0–18.7	11
18.7–20	12

Table 7-8. Harmonic Selection for Moderate Walking (1.7–2.0 Hz)	
Dominant Frequency, Hz	***h***
8–10	5
10–12	6
12–14	7
14–16	8
16–18	9
18–20	10

Table 7-9. Harmonic Selection for Fast Walking (2.0–2.2 Hz)	
Dominant Frequency, Hz	***h***
8.8–11	5
11–13.2	6
13.2–15.4	7
15.4–17.6	8
17.6–20	9

7.5.4 Design Examples

Example 7.3—Floor Supporting Sensitive Equipment

Given:

The floor system shown in Figure 7-19 is to be evaluated for vibration at the equipment location shown due to walking in the adjacent corridor. The equipment tolerance is a one-third octave spectral velocity limit of 4,000 mips. The total slab thickness is 6¼ in. with w_c = 110 pcf, f'_c =3 ksi concrete on 2-in.-thick deck weighing 2 psf. Uniform superimposed dead load is 8 psf, and the live load is 8 psf. The spandrel beams and girders support cladding weighing 720 plf. The beams and girders are ASTM A992 material.

The plan shows the full-height (16-ft) partitions supported by the slab, partition weight = 5.5 psf. These extend up to typical slip tracks at the underside of the floor above. Partitions will also be installed below but are ignored because they might be removed or significantly modified in the future. The supported partitions are to be modeled with vertical springs with 2.0 kip/in./ft stiffness.

Uniform Mass

$$\text{Slab concrete} = (110\text{ pcf})\left(\frac{5.25\text{ in.}}{12\dfrac{\text{in.}}{\text{ft}}}\right)$$
$$= 48.1\text{ psf}$$

Deck = 2 psf
Mechanical, electrical, plumbing and ceiling = 8 psf
Live load = 8 psf

$$\text{Total supported load} = 48.1\text{ psf} + 2.00\text{ psf} + 8.00\text{ psf} + 8.00\text{ psf}$$
$$= 66.1\text{ psf}$$
$$\text{Total supported load to the shells} = \frac{66.1\text{ psf}}{32.2\text{ ft/s}^2}$$
$$= 2.05\text{ psf-s}^2\text{/ft}$$

Cladding:

$$\text{Line mass} = \frac{720\text{ plf}}{32.2\text{ ft/s}^2}$$
$$= 22.4\text{ plf-s}^2\text{/ft}$$

Member masses:

Computed by the program.

Partitions:

$$\text{Line mass along the wall} = \frac{(5.50\text{ psf})(16.0\text{ ft})}{32.2\text{ ft/s}^2}$$
$$= 2.73\text{ plf-s}^2\text{/ft}$$

Nodes are approximately 3.5 ft apart; therefore, (2.73 plf-s^2/ft)(3.50 ft) = 9.56 lb-s^2/ft is applied at each node.

Damping

Viscous damping of 5% of critical is assumed because there are multiple full-height partitions in most bays.

Slab Stiffness

$1.35E_c$ = 2,700 ksi
Moment of inertia parallel to the deck = 176 in.^{4}/ft
Moment of inertia perpendicular to the deck = 76.7 in.^{4}/ft

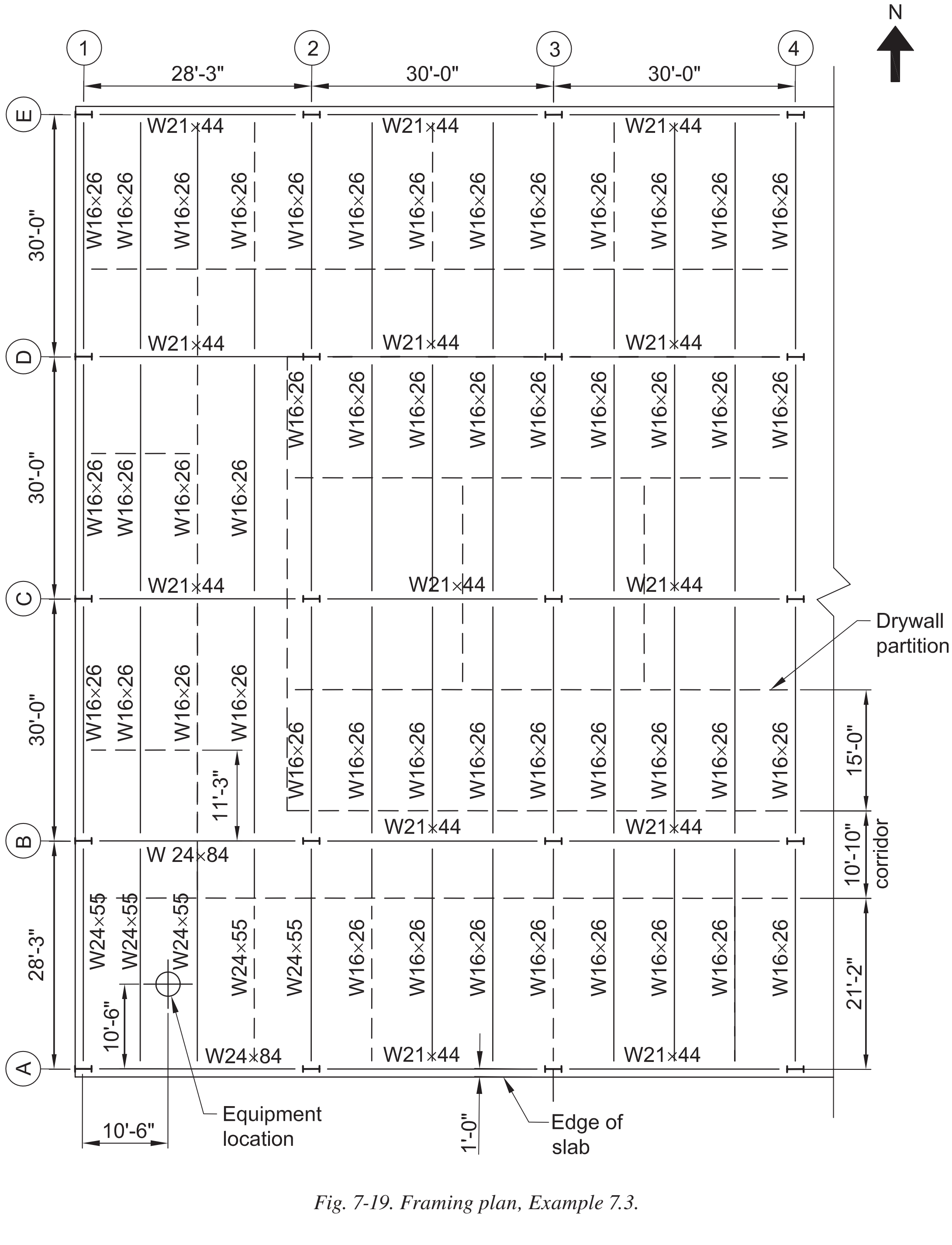


Fig. 7-19. Framing plan, Example 7.3.

Member Transformed Moments of Inertia

The following transformed moments of inertia are computed as described in Section 7.2:

W16×26 typical beam: 1,210 in.^{4}
W21×44 typical girder: 2,960 in.^{4}
W24×55 beam in bay A-1/B-2: 4,120 in.^{4}
W24×84 interior girder in bay A-1/B-2: 6,740 in.^{4}
W24×55 spandrel beam in bay A-1/B-2: 9,230 in.^{4} (includes 2.5 times increase due to cladding restraint)
W24×84 spandrel girder in bay A-1/B-2: 14,600 in.^{4} (includes 2.5 times increase due to cladding restraint)

Model Development

The model shown in Figure 7-20 was created in a commercially available structural analysis program. The model is terminated at Gridline 4, even though the floor extends several additional bays to the east. All members are modeled as having flexurally continuous connections. Cladding line mass is assigned to each member along Gridlines 1, A and E. Uniform mass is assigned to shells and partition masses are assigned to nodes along the partition walls as shown in Figure 7-21.

Natural Mode Prediction

Numerous natural modes—16 between 5.5 Hz and 10 Hz and 50 below 20 Hz—are predicted using standard eigenvalue analysis, and it is unclear which causes high responses at the equipment. Thus, the FRF magnitude is used to select the critical modes.

Frequency Response Function Prediction

The FRF for acceleration at the equipment location due to walking in the corridor (locations shown in Figure 7-22) was computed. The load (walking) location is as close to the middle of Bay A-1/B-2 as is practically possible considering the floor plan. Figure 7-23 shows the predicted FRF magnitude, which indicates responsive natural modes at 10.9 Hz and 16.0 Hz. The former

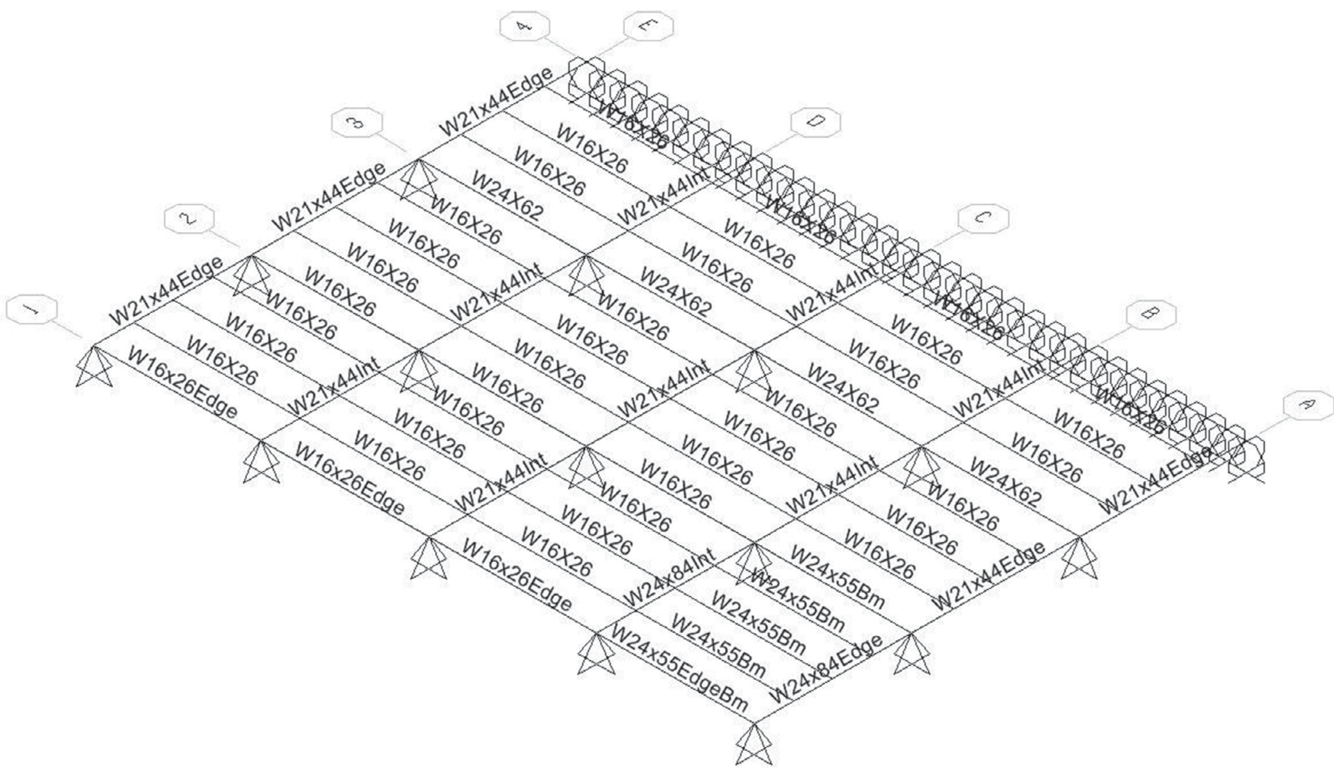


Fig. 7-20. Finite element model, Example 7.3.

has a lower frequency and higher magnitude; thus, it will produce the highest response if excited. The 10.9-Hz dominant mode shape is shown in Figure 7-24.

Predicted Velocity Due to Walking in Corridor

Fast walking is possible in the corridor. From Table 7-9, the fifth harmonic frequency can equal 10.9 Hz, and the step frequency is

$$
\begin{aligned}
f_{step} &= \frac{f_n}{h} \\
&= \frac{10.9 \text{ Hz}}{5} \\
&= 2.18 \text{ Hz}
\end{aligned}
$$

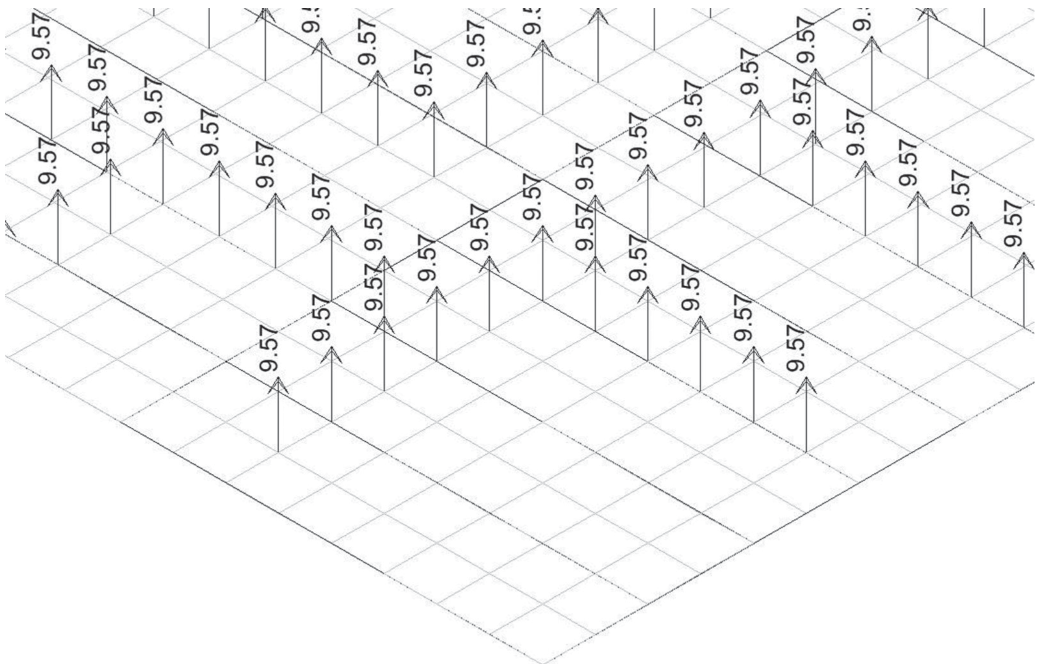


Fig. 7-21. Partition mass (lb-s²/ft) assignment, Example 7.3.

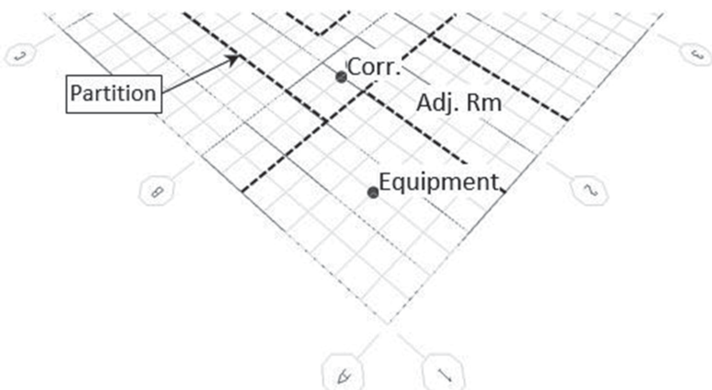


Fig. 7-22. Node labels at equipment and corridor, Example 7.3.

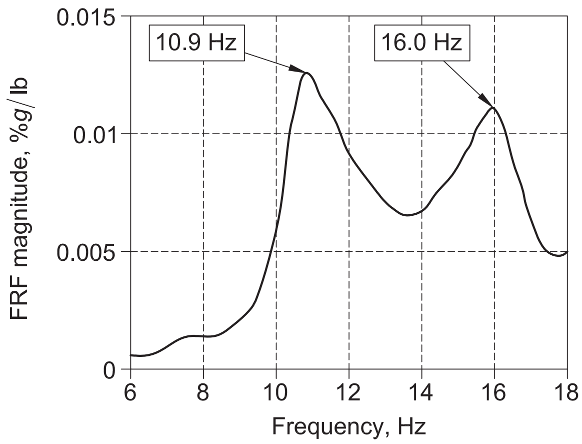


Fig. 7-23. Predicted FRF magnitude, Example 7.3.

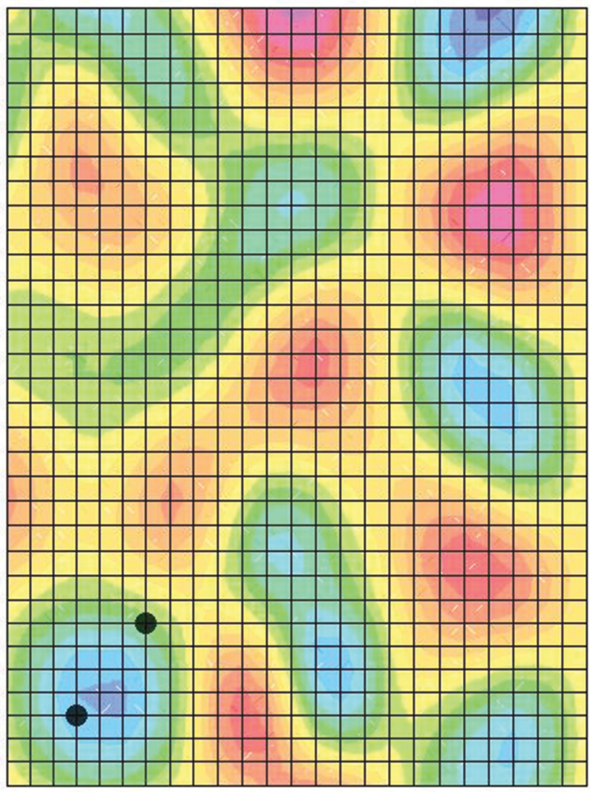


Fig. 7-24. Predicted dominant mode shape (10.9 Hz), Example 7.3.

Table 7-10. Computed Natural Frequencies and Mode Shapes, Example 7.3							
Mode, *m*	$f_{n,m}$, Hz	ϕ_{Corr}, $(\text{in./kip-s}^2)^{0.5}$	ϕ_{Equip}, $(\text{in./kip-s}^2)^{0.5}$	Mode, *m*	$f_{n,m}$, Hz	ϕ_{Corr}, $(\text{in./kip-s}^2)^{0.5}$	ϕ_{Equip}, $(\text{in./kip-s}^2)^{0.5}$
1	5.54	0.0645	0.0126	26	12.8	0.340	–0.981
2	5.79	0.303	0.151	27	13.1	–0.218	0.272
3	6.29	0.268	0.228	28	13.2	–0.0255	–0.741
4	6.76	0.0397	0.00555	29	13.5	–0.303	0.698
5	6.91	0.0283	–0.0421	30	13.8	0.181	–0.454
6	6.97	0.0560	0.126	31	13.9	–0.247	–0.0667
7	7.11	0.101	0.0862	32	14.0	–0.136	–0.937
8	7.38	0.468	0.131	33	14.4	0.154	–0.446
9	7.77	0.178	0.235	34	14.7	–0.104	0.183
10	7.80	0.522	0.3290	35	15.1	0.345	–0.661
11	8.28	–0.315	–0.373	36	15.3	–0.0923	0.174
12	8.44	0.0715	0.302	37	15.6	–0.0767	0.0187
13	8.95	0.231	0.624	38	15.9	0.00420	0.120
14	9.27	–0.0135	0.0185	39	16.1	1.29	–2.61
15	9.56	–0.0807	0.0676	40	16.4	0.365	–0.394
16	9.90	–0.268	0.153	41	16.7	0.0917	–0.0305
17	10.2	0.375	0.105	42	16.9	0.451	–0.430
18	10.5	–0.452	–0.223	43	17.0	0.150	–0.295
19	10.7	–1.46	–2.51	44	17.2	–0.312	–0.180
20	10.9	0.558	1.29	45	18.1	–0.0890	0.485
21	11.3	0.513	1.72	46	18.2	–0.232	1.16
22	11.7	–0.298	–0.113	47	18.3	–0.581	0.734
23	12.0	0.341	0.057	48	18.6	–0.422	0.572
24	12.1	0.0309	–0.324	49	18.8	–0.196	0.859
25	12.2	0.272	–0.410	50	19.5	–0.493	1.20

The response of each mode is computed using Equations 7-20 and 7-21. The mode shape values, ϕ_i and ϕ_j, at the corridor and equipment nodes, respectively, are shown in Table 7-10. The program reports mass-normalized mode shapes with $(\text{in./kip-s}^2)^{0.5}$ units.

For example, the Mode 19 response is

$$I_{eff,19} = \left(\frac{f_{step}^{\,1.43}}{f_{n,19}^{\,1.30}}\right)\left(\frac{Q}{17.8}\right) \quad \text{(from Eq. 1-6)}$$

$$= \left[\frac{(2.18\text{ Hz})^{1.43}}{(10.7\text{ Hz})^{1.30}}\right]\left(\frac{168\text{ lb}}{17.8}\right)$$

$$= 1.32\text{ lb-s}$$

$$a_{p,19} = 2\pi f_{n,19}\phi_{i,19}\phi_{j,19}I_{eff,19} \qquad \text{(from Eq. 7-4)}$$

$$= 2\pi(10.7\text{ Hz})\left(-1.46\sqrt{\text{in./kip-s}^2}\right)\left(-2.51\sqrt{\text{in./kip-s}^2}\right)\left(\frac{1.32\text{ lb-s}}{1{,}000\text{ lb/kip}}\right)\left(\frac{100\%\text{g}}{386\text{ in./s}^2}\right)$$

$$= 0.0843\%g$$

$$a_{19}(t) = a_{p,19}e^{-2\pi f_{n,19}\beta t}\sin(2\pi f_{n,19}t) \qquad (7\text{-}5)$$

$$= (0.0843\%g)e^{-2\pi(10.7\text{ Hz})(0.05)(t)}\sin[2\pi(10.7\text{ Hz})t]$$

Similarly, the responses for the remaining modes were computed and then summed to obtain the total response. Using a footstep period, T_{step}, of $1/f_{step} = 0.459$ s, Mode 19 and total responses are shown in Figure 7-25. The peak uncalibrated acceleration is $0.156\%g$ at 0.075 s. The calibrated peak, a_p, is $1.5(0.156\%g) = 0.234\%g$.

Using $h = 5$ for the fifth harmonic frequency noted earlier, the predicted narrowband spectral acceleration maximum magnitude is, then,

$$A_{NB} = a_p\frac{1-e^{-2\pi\beta h}}{20\beta h} \qquad (7\text{-}23)$$

$$= (0.234\%g)\frac{1-e^{-2\pi(0.05)(5)}}{20(0.05)(5)}$$

$$= 0.0371\%g$$

Finally, the predicted one-third octave spectral velocity is

$$V_{1/3} = 0.8\frac{A_{NB}}{2\pi}\sqrt{\frac{T}{30f_n}} \qquad (7\text{-}24)$$

$$= 0.8\left(\frac{0.0371\%g}{2\pi}\right)\left(\sqrt{\frac{8.00\text{ s}}{30(10.9\text{ Hz})}}\right)\left(\frac{386\text{ in/s}^2}{100\%g}\right)\left(\frac{10^6\text{ mips}}{\text{in./s}}\right)$$

$$= 2{,}850\text{ mips}$$

The predicted response does not exceed the 4,000-mips tolerance limit; therefore, the evaluation criterion is satisfied.

Note that fast walking in the corridor is shown for illustration of the evaluation process. In an actual design, walking in the adjacent room should also be evaluated.

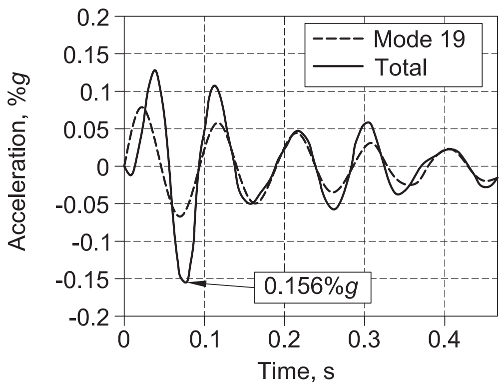


Fig. 7-25. Mode 19 and total responses, Example 7.3.

Chapter 8
Evaluation of Vibration Problems and Remedial Measures

This chapter provides guidance on vibration evaluation and on remedial measures to resolve floor vibration problems that can arise in existing buildings. Occupant vibration complaints due to walking are usually associated with open office or residential fit-outs and relatively large spans. Failure to check a floor for vibration tolerance or overestimation of damping by the designer (e.g., assuming that the fit-out will include fixed partitions or paper office fit-out) are typical causes of a problem or lively floor. There have also been a number of cases reported where a new tenant has removed fixed partitions in an older building to allow an open floor plan, resulting in occupant vibration complaints because of the reduced damping. Change of occupancy use, such as the introduction of a health club or heavy reciprocating machinery, or installation of sensitive equipment, can result in problems as well.

Occupant complaints due to rhythmic activities are usually associated with quiet occupant spaces adjacent to or above the activity area. There have been a number of instances where aerobic type activity on a lower level of a multistory building caused objectionable vibrations at an upper level (Allen, 2012; Lee et al., 2013).

Sensitive equipment vibration problems are generally associated with floors that are not sufficiently massive and stiff to support the equipment.

8.1 EVALUATION

Source Determination

It is important, first of all, to determine the source of vibration, be it walking, rhythmic activities, equipment, or sources external to the building that transmit vibration through the ground.

Evaluation Approaches

Possible evaluation approaches are:

- performance tests
- calculations
- vibration measurements

A performance test is particularly useful prior to a change in use of an existing floor. For example, the effect of a contemplated use of a room for aerobics can be evaluated by having typical aerobics performed while people are located in sensitive occupancies to observe the resulting vibration. Simple walking tests with a few people placed at potential sensitive locations can be carried out for floor areas contemplated for office, residential or other sensitive occupancies. A resonant harmonic step frequency should be used for the testing.

Calculations as described in Chapters 3 through 7 can be used to evaluate the dynamic properties of a structure and to estimate the vibration response caused by dynamic loading from human activities. Calculations, however, may be associated with significant uncertainties and therefore testing is preferable when possible.

Measurements can be used to evaluate the dynamic properties of a structure, as well as to quantify the vibrations associated with human activities. Long-term event monitoring using accelerometers and triggered recording instrumentation is generally not useful for this purpose because the source of the event that caused the triggering will not be known unless video recording is available. Measurement of floor motion due to random walking or an arbitrary walking speed does not capture the maximum possible amplitude of the vertical motion and is of little use in evaluating a floor. A recommended procedure for testing floors where occupant complaints have been received, because of walking, is found in the following section.

8.2 RECOMMENDED VIBRATION MEASUREMENT TECHNIQUES

Walking Testing and Evaluation

Most complaints because of vibrations due to walking are associated with low-frequency floors. A low-frequency floor—i.e., a floor with a natural frequency less than about 9 Hz—is susceptible to resonance with one of the first four harmonics of the walking force. When complaints are received, it is likely that at least one occupant's natural walking gait matches a subharmonic of the natural frequency causing occasional resonant build-ups. In fact, the walking of one occupant, whose natural gait is at the subharmonic of the floor natural frequency, is almost always the source of lively floor complaints.

To experimentally determine the largest floor response, walking tests should be done at a step frequency at a subharmonic of the floor dominant frequency and between approximately 96 steps/min and 132 steps/min (1.6 Hz and 2.2 Hz), which is the normal range of human walking speeds on a flat surface. For example, if the dominant frequency is

6 Hz, then the walking should be at 120 steps/min (2.0 Hz), so the third harmonic causes resonance. The second harmonic was not chosen because the step frequency to cause resonance would have to be 6.00 Hz/2 = 3.00 Hz, which is outside the normal step frequency range. The fourth harmonic was not chosen, even though 6.00 Hz/4 = 1.50 Hz, which is nearly within the normal range of human walking, because the fourth harmonic force amplitude is lower than that of the third harmonic. (See Figure 2-4.)

The most accurate current experimental method is shaker-based experimental modal analysis (EMA) as described in Ewins (2000) and Barrett (2006). EMA is used to fully define the natural modes and resonant frequencies of the floor and is the most accurate method for determining dumping in a structure. EMA also provides information useful for finite element model tuning, which may be needed for retrofit design. Accelerations are then recorded during walking at resonant frequencies for comparison with tolerance limits and to provide additional information for finite element model tuning. There are three impediments to using EMA: (1) shaker-based EMA testing equipment is prohibitively expensive for many structural engineering firms; (2) the cost and time required to mobilize, perform tests and process data may be prohibitive; and (3) the disruption caused by shaker-based testing is often unacceptable to the client.

Davis et al. (2014) developed a simplified testing procedure and have used it extensively to successfully evaluate lively floor motions and to gather information useful for developing retrofit schemes. The only instrumentation needed is a handheld, single-channel, spectrum analyzer and a seismic accelerometer, requiring no shipping costs and allowing very rapid mobilization, testing and data processing. The evaluation begins with estimation of natural frequencies using simple heel-drop tests. The maximum sinusoidal response due to walking is then determined for comparison with human comfort limits with the walking speed previously described. A metronome is used by the walker to aid in matching the required step frequency.

Because the tolerance limits recommended in Chapter 2 are in terms of sinusoidal peak accelerations, a single peak in a measured waveform is not directly comparable. For a true comparison, the equivalent sinusoidal peak acceleration (ESPA) is determined. First, the recorded acceleration versus time waveform is low-pass filtered to exclude high-frequency content (above 15 to 20 Hz) to which humans are insensitive. Then 1- or 2-second root-mean-square (RMS) accelerations are computed. Davis et al. (2014) used a 2-second increment, whereas, ISO 2631-2 (ISO, 1989) recommends a 1-second increment. The latter is also known as the maximum transient vibration value, or MTVV. The maximum running RMS acceleration is converted to ESPA by multiplying it by $\sqrt{2}$, which is the ratio of peak-to-RMS acceleration for a sinusoid. The ESPA, in $\%g$, is then compared to the appropriate acceptance criterion in Chapter 2.

Once the floor natural frequency and ESPA are determined, remedial measures can be designed. Typically, FEA methods from Chapter 7 are required to determine the adequacy of the proposed solution. It is possible that this procedure shows that the floor, in fact, satisfies the acceptance criteria, and a retrofit is not necessary.

Rhythmic Testing and Evaluation

Rhythmic testing and evaluation may be needed to determine the magnitude of objectionable floor motion or to verify that occupant activities are the cause of the vibrations in areas remote from the activity floor. The latter is particularly important when objectionable vibrations are reported elsewhere in the building. Ambient or heel-drop records can be used to determine the natural frequency of the activity floor. The peak response can then be determined by performing a typical rhythmic activity at a harmonic frequency with the aid of a metronome. To determine if activities on the floor in question are causing motion at other locations, even many floors above or below the activity level, recording motion at the annoyance location and comparing the time of the event, duration and frequencies at the activity level can be done.

8.3 REMEDIAL MEASURES

Reduction of Effects

In some situations, it may suffice to do nothing about the structural vibration itself but to use measures that reduce the annoyance associated with the vibration. This includes the elimination of objectionable vibration cues, such as noise due to rattling, and removing or altering furniture or non-structural components that vibrate in resonance with the floor motion.

Relocation

The vibration source (e.g., aerobics, reciprocating equipment) and/or a sensitive occupancy or sensitive equipment may be relocated. It is obviously preferable to do this before the locations are finalized. For example, a planned aerobics exercise facility might be relocated from the upper floor of a building to a ground floor or to a stiff floor. Complaints about walking vibration can sometimes be resolved by relocating one or two sensitive people, activities or equipment items, e.g., placing these near a column where vibrations are less severe than at midbay.

Reducing Mass

Reducing the mass is usually not very effective because of the resulting reduced inertial resistance to impact or to resonant vibration. Occasionally, however, reducing the mass can increase the natural frequency sufficiently so that resonance is avoided.

Stiffening

Vibrations due to walking or rhythmic activities can be reduced by increasing the floor natural frequency. This is best done by increasing structural stiffness. Retrofitting with proper testing, jacking, welding, and stabilizing out-of-plane motion is absolutely necessary for achieving significant stiffness increases. Jacking is required to introduce strain into the added elements so that the retrofitted system is "tight" and able to respond to the very small loads introduced by human activity.

The structural components with the lower fundamental frequencies are usually the ones that should be stiffened. For low dynamic loading, such as walking, an evaluation of the floor structural system considering only the girders and joists or beams usually suffices. For severe dynamic loading (e.g., rhythmic exercises or heavy equipment), the evaluation must consider the building structure as a whole, including the columns and possibly the foundations, not just the floor structure.

Some examples of stiffening are shown in Figure 8-1. New column supports down to the foundations between existing ones are most effective for flexible floor structures, Figure 8-1(a), but often this approach is not acceptable to the owner. A damping element, such as a friction device or one using visco-elastic material, may absorb some vibrational energy. The benefit of damping posts is limited to, at most, the effective width of the joist panel (see Chapter 4 for joist panel width). Usually, one column or post per bay is required.

Stiffening the supporting beams and girders by adding cover plates or rods as shown in Figure 8-1(b) is not particularly effective, even if the floor system is jacked up prior to welding. The addition of rods to the bottom chord of joists is also not very effective. Even with jacking, the expected increase in frequency generally does not occur because only the flexural stiffness of the joist is increased, while the effect of deformation due to shear and eccentricity at joints (see Sections 3.5 and 3.6) may be increased. Adding a hot-rolled section as shown in Figure 8-1(b) may be effective if the floor is raised by jacking prior to welding the section. If this type of stiffening is used for open web joists or joist girders, the effects on shear and eccentricity on stiffness need to be evaluated using the finite element method.

Installation of a light concrete masonry unit (CMU) wall in place of typical non-loadbearing drywall partitions has been used to successfully stiffen girders.

A technique that has been shown to be effective if there is enough ceiling space is to weld a queen post hanger to the bottom flange of a beam or joist chord as shown in Figure 8-1(c). This arrangement substantially increases the member stiffness. The hanger can be placed around existing ducts and pipes in the ceiling space. Repairs can be carried out at night or on weekends by temporarily removing ceiling tiles below each member to be stiffened. Jacking up the floor before welding is required to introduce stress in the hanger as would occur if the hanger was installed during initial construction. Lateral support of the new horizontal member is required and must be installed before the floor is lowered.

If the supporting member is separated from the slab—e.g., in the case of overhanging beams which pass over a supporting girder or joist seats supported on the top flange of a girder—the girder can be stiffened as shown in Figure 8-1(d). Generally, two to four pieces of the overhanging beam section or a section of equivalent depth, placed with their webs in the plane of the web of the girder and attached to both the slab and girder, provide sufficient shear connection for composite action between the slab and the girder. Similarly, composite action may be achieved for girders supporting joist seats by installing short sections of the joist seat profile, an HSS, or similar section as shown in Figure 8-1(d). For both cases, the supporting girder must be jacked up prior to installation of the beam or joist seat shear connectors.

Adding bridging to systems with concrete slabs in an attempt to improve vibration performance has not been found to be successful. However, bridging has been used successfully to reduce vibration of an exterior patio area with a very flexible deck and roof paver system (Davis et al., 2014).

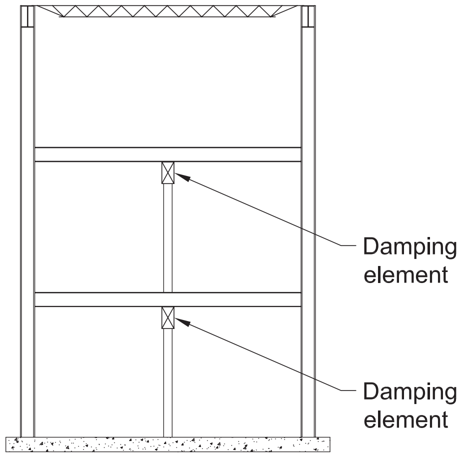
Sometimes, the troublesome vibration mode involves flexure of vertical members (e.g., structural framing with cantilevers from columns or walls), in which case, both horizontal and vertical stiffening will be required. In these situations, it is important to know the shape of the troublesome mode.

Damping Increase with Nonstructural Elements

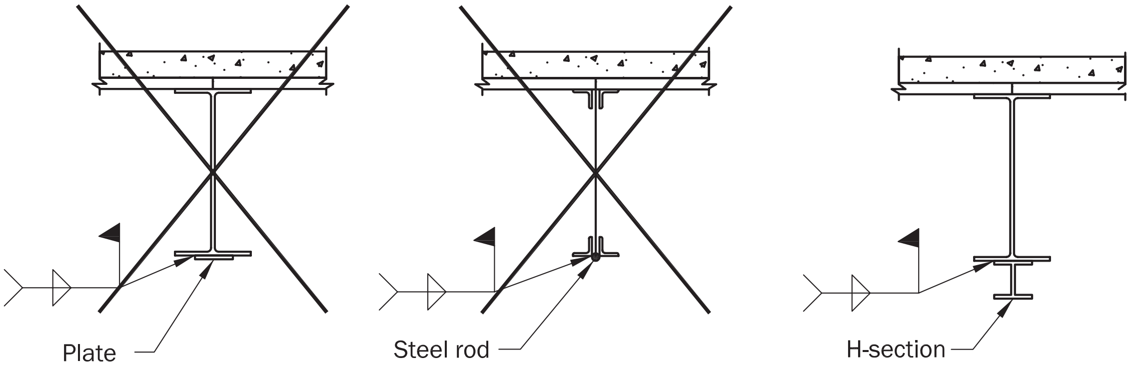
Floor vibrations can be improved by increasing the damping of the floor system. The smaller the damping is in the existing floor system, the more effective is the addition of damping. Damping in existing floors depends primarily on the presence of nonstructural components, such as partitions, ceilings, mechanical service lines, furnishings, and on the number of people on the floor.

Passive Control

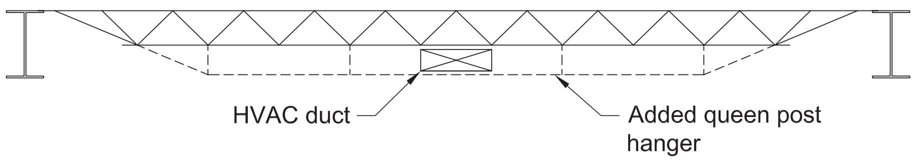
Passive control of floors in the form of a tuned mass damper (TMD) has been used with varying degrees of success. A TMD is a mass attached to the floor structure through a spring and damping device. The TMD prevents build-up of resonance vibration of a floor by transfer of kinetic energy from the floor into the TMD mass and dissipating some of this kinetic energy via the damping devices. A TMD is effective, however, only if the natural frequency of the TMD



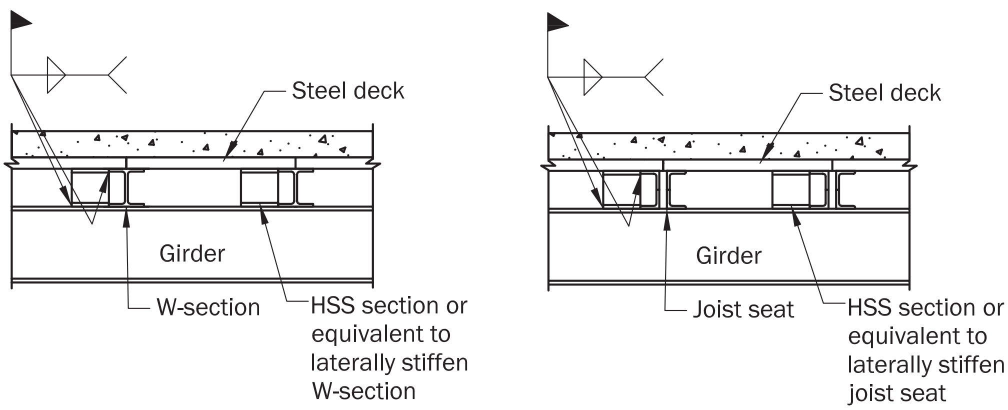
(a) Post with or without damping element



(b) Bottom chord reinforcing



(c) Queen post stiffening



(d) Support stiffening

Fig. 8-1. Methods for stiffening floors.

nearly matches that of the troublesome mode of floor vibration. The effectiveness of a TMD tuned to the troublesome mode of vibration can be estimated from the effective damping ratio of the floor-TMD system using

$$\beta_d = 0.5\sqrt{m/M} \tag{8-1}$$

where
M = fundamental modal mass of floor, lb-s^2/in.
m = mass of the TMD, lb-s^2/in.
β_d = damping added by the TMD

Thus, if the mass ratio, m/M, is equal to 0.01, the effective damping ratio is 0.05. This can result in a considerable reduction in resonant vibration for a lightly damped floor or footbridge but little reduction for a floor with many partitions or many people on it, which already is relatively highly damped.

TMDs are most effective if there is only one significant mode of vibration (Bachmann and Weber, 1995; Webster and Vaicajtis, 1992). They are much less effective if there are two or more troublesome modes of vibration whose natural frequencies are close to each other (Murray, 1996). They are ineffective for off-resonance vibrations, which can occur during rhythmic activities. Finally, TMDs that are initially tuned to floor vibration modes can become out-of-tune due to changes in the natural frequencies of the floor resulting from the addition or removal of materials in local areas.

To be effective for vibrations from aerobics, the mass of a TMD must usually be much greater than for walking vibration. This is because the system damping ratio, β_s, usually must be much greater to reduce aerobics vibrations in the building to acceptable levels in sensitive occupancies. The people on the floor, including the participants, already provide significant damping to the floor system. TMDs have sometimes proven successful when the effective floor mass is large relative to the number of participants and if the acceleration at resonant vibrations is less than approximately 10% gravity.

Floating Floor for Rhythmic Activities

An effective method for reducing building vibration due to machinery is to isolate the machinery from the building by placing the machine on soft springs. This concept can also be used for rhythmic activities by inserting a "floating floor" mass on very soft springs between the participants and the building floor supporting the activity. This idea is attractive for rhythmic activities in the upper stories of buildings because it avoids the need to greatly stiffen the building structure, and the floating floor can be introduced when it is needed on an existing floor area and removed when it is no longer required. The increased loading due to the floating floor is offset, at least partly, by the reduced live load transmitted to the building floor. However, spring elements that are soft enough to isolate rhythmic activities considerably are often impractical.

Reduction of Vibration Transmission

Extremely objectionable floor vibrations sometimes occur in large, open floor areas where the floor is supported by identical, equally and closely spaced joists or beams, as shown in Figure 8-2. The response of the floor due to a heel-drop type impact, measured 65 ft from the heel-drop location, is shown in Figure 8-3. This type of response with a "beat" (periodic change in amplitude) of 1 to 2 seconds, is particularly objectionable. The sensation is "wave-like" with waves rolling back and forth across the width of the building. Also, because of the transmission of the vibration, an occupant who is unaware of the cause of the motion is suddenly subjected to significant motion and may be particularly annoyed.

Vibration transmission of this type can be reduced, if not eliminated, by changing the stiffness of some of the joist members—say, at the column lines—or by changing the spacing in alternate bays. In a completed structure, stiffening of joists at columns may be a practical way to reduce vibration transmission significantly. (Note, stiffening a joist or several joists within a bay has been shown not to be effective.)

8.4 PROTECTION OF SENSITIVE EQUIPMENT

Remedial measures for reducing the exposure of sensitive equipment to vibrations induced by walking include relocation of equipment to areas where vibrations are less severe, providing vibration isolation devices for the equipment of concern, or implementing structural modifications that reduce the vibrations of floors that support the sensitive equipment. Some of the relevant issues are discussed in Section 6.1.7.

Equipment that is subject to excessive vibration may benefit from being moved to locations near columns. It is beneficial to move such equipment to bays in which there are no corridors and that are not directly adjacent to corridors—particularly, to heavily traveled corridors. The most favorable locations for sensitive equipment are typically at grade (i.e., on the ground), but on suspended floors, the best locations are those that are as far as possible from areas where considerable foot traffic can occur.

Vibration isolation devices are readily available for many items of sensitive equipment. These devices are typically resiliently supported platforms, tables or cradles; the resilient supports generally consist of arrangements of steel springs, rubber elements or air springs. Isolation systems are often available from the equipment manufacturers and generally can be obtained from suppliers who specialize in vibration isolation. Because selection and/or design of vibration isolation for sensitive equipment involves a number of mechanical considerations and engineering trade-offs, it usually is best left to specialists. Structural modifications that reduce the vibrations of floors on which sensitive equipment is located include stiffening of the floors of the bays in which the equipment is situated, separating these bays from corridors in which significant walking occurs by the introduction of joints, or providing walk-on floors that do not communicate directly with the floors that support the sensitive equipment. Such floors might be floated on soft isolation systems or may be supported only at the columns, for example.

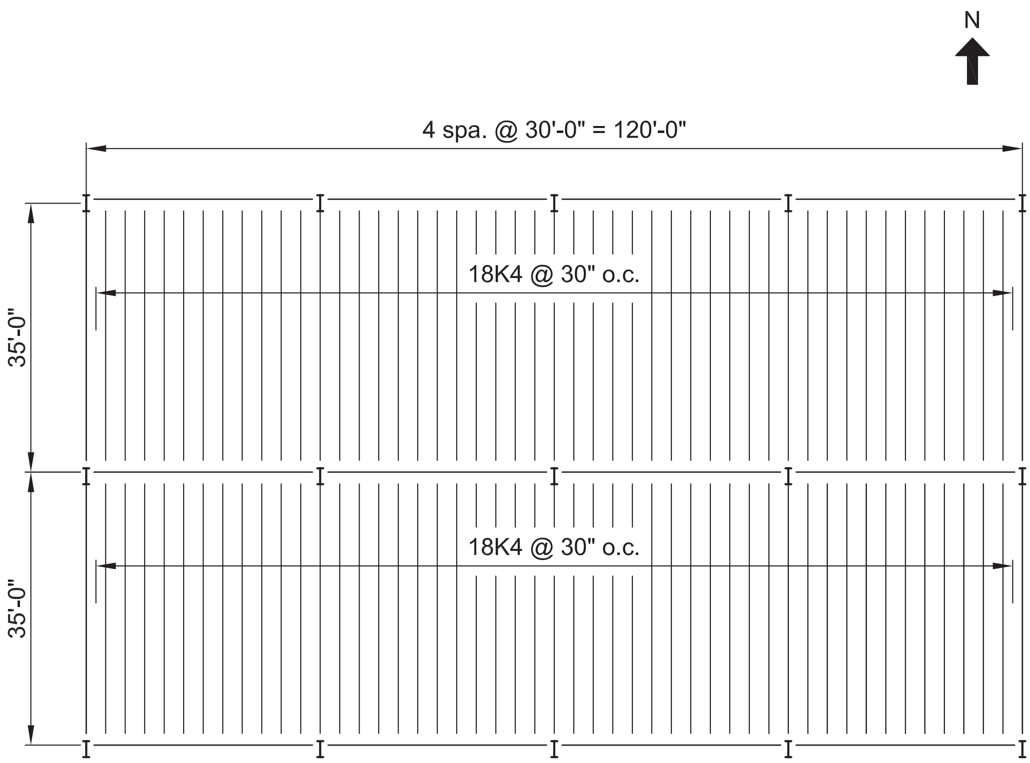


Fig. 8-2. Large open area supported by equally spaced joists.

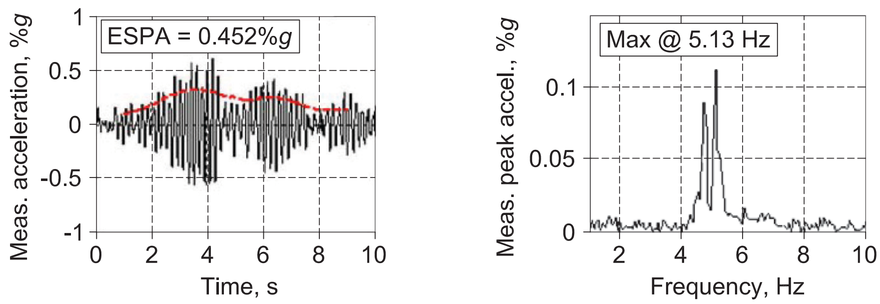


Fig. 8-3. Floor response with "beat."

SYMBOLS

$A_{1/3}$ maximum one-third octave spectral acceleration due to walking, in./s^2

$A_{1/3,Lim}$ equipment tolerance limit expressed as one-third octave spectral acceleration, in./s^2

A_{NB} maximum narrowband spectral acceleration due to walking, in./s^2

$A_{NB,Lim}$ equipment tolerance limit expressed as narrow-band spectral acceleration, in./s^2

B effective panel width, ft

B_g girder panel mode effective width, ft

B_j joist panel mode effective width, ft

C_g 1.6 for girders supporting joists connected to the girder flange with joist seats; 1.8 for girders supporting beams connected to the girder web

C_j 2.0 for joists or beams in most areas; 1.0 for joists or beams parallel to a free edge (edge of balcony, mezzanine, or building edge if cladding is not connected)

C_r Equation 3-9

D nominal depth of joist or joist girder, in.

D_g girder transformed moment of inertia per unit width, in.^{4}/ft

D_j joist or beam transformed moment of inertia per unit width, I_j/S, in.^{4}/ft

D_s slab transformed moment of inertia per unit width, in.^{4}/ft

E_c modulus of elasticity of concrete = $w^{1.5}\sqrt{f_c'}$, ksi

E_s modulus of elasticity of steel = 29,000 ksi

EI_t stringer vertical flexural stiffness, including stringers and any other elements that provide stiffness, kip-in.^{2}

ESD energy spectral density at frequency

$E_{1/3}$ energy attributed to all frequencies in a one-third octave band

$F(t)$ human-induced force as a function of time, lb

FRF_{Max} maximum FRF magnitude, %g/lb

G_c elastic shear modulus of concrete calculated using 1.35E_c, ksi

I_{chords} moment of inertia of the chord areas, in.^{4}

I_{comp} fully composite transformed moment of inertia of the slab and chord areas, in.^{4}

I_{comp} fully composite moment of inertia of the slab and girder areas, in.^{4}

I_e effective transformed moment of inertia of the truss accounting for shear deformation, in.^{4}

I_e effective moment of inertia of the joist or joist girder accounting for shear deformations and joint eccentricities, in.^{4}

I_{eff} effective impulse, lb-s

$I_{eff,m}$ effective impulse computed for mode m, lb-s

I_g transformed or effective moment of inertia of the girder or joist girder, in.^{4}

I_j transformed or effective moment of inertia of the beam or joist, in.^{4}

I_t transformed moment of inertia, in.^{4}

I_t effective transformed moment of inertia if shear deformations are included, in.^{4}

I_t reduced transformed moment of inertia to account for joist seat flexibility, in.^{4}

I_x moment of inertia of the girder, in.^{4}

L joist or joist girder span; member span, in.

L_g girder span, ft

L_j joist or beam span, ft

L_s stair stringer length measured along the diagonal between supports, in.

M fundamental modal mass of the system, lb-s^2/in.

M_s fundamental modal mass of the stair, lb-s^2/in.

N number of considered harmonics, i.e., the number of harmonics with significant amplitudes

N number of discrete acceleration data points between one footstep and the next

N_{modes} number of modes

N_{Steps} number of footsteps

P amplitude of the forcing driving force, lb

P amplitude of sinusoidal load, lb

P_o amplitude of the driving force, lb

$P(t)$ uniform dynamic load for rhythmic activities, psf

Q bodyweight, lb

R calibration factor

R reduction factor

R_M higher mode factor, 2.0

S joist or beam spacing, ft

T footstep period ($1/f_{Step}$), s

T walking event duration, taken as 8 s

T_{BU} resonant build-up duration, s

T_{step} footstep period equal to $1/f_{Step}$, s

$V(f)$ narrowband spectral velocity at frequency f, mips

$V(f_{ctr})$ spectral velocity at one-third octave band centered at f_{ctr}, mips

$V_{1/3}$ maximum one-third octave spectral velocity due to walking, mips

$V_{1/3,\,Lim}$ equipment tolerance limit expressed as one-third octave spectral velocity, mips

V_{NB} maximum narrowband spectral velocity due to walking, mips

$V_{NB,Lim}$ equipment tolerance limit expressed as narrowband spectral velocity, mips

W effective weight of the floor bay or weight of the pedestrian bridge, lb

W effective weight supported by the beam or joist panel, girder panel or combined panel, lb

W effective combined mode panel weight, lb

W_g effective girder panel weight, lb

W_j effective joist or beam panel weight, lb

W_s weight of stair, lb

$a(t)$ acceleration as a function of time, t, in./s^2

a_{ESPA} equivalent sinusoidal peak acceleration, in./s^2

a_k kth acceleration data point, in./s^2

a_o acceleration tolerance limit, in./s^2

a_p peak acceleration, in./s^2

$a_{p,i}$ peak acceleration due to harmonic, i, in./s^2

$a_{p,Lim}$ equipment tolerance limit expressed as peak acceleration, in./s^2

$a_{p,m}$ peak acceleration due to mode m, in./s^2

$a_{SteadyState}$ steady-state acceleration, in./s^2

a_{RMS} root-mean-square acceleration, in./s^2

a_o/g vibration tolerance limit expressed as an acceleration ratio

$a_{p,i}/g$ peak acceleration ratio for harmonic i as a fraction of the acceleration due to gravity

a_p/g ratio of the peak floor acceleration to the acceleration of gravity

d_e effective depth of the concrete slab, taken as the depth of the concrete above the deck plus one-half the depth of the deck, in.

f_1 one-third octave band lower limit, Hz

f_2 one-third octave band upper limit, Hz

f_{4max} fourth harmonic maximum frequency, Hz

f_L intermediate zone lower boundary, Hz

f_U intermediate zone upper boundary, Hz

f_g girder panel mode frequency, Hz

f_j beam or joist panel mode frequency, Hz

f_n fundamental natural frequency, Hz

f_n dominant frequency, Hz

f_n floor natural frequency, Hz

$f_{n,m}$ natural frequency of mode, Hz

f_{step} step frequency, Hz

g acceleration of gravity = 386 in./s^2

h number of the harmonic that causes resonance

h step frequency harmonic matching the harmonic number

i harmonic number

k floor stiffness, kip/in.

k partition spring stiffness, kip/in.

m mass of the TMD, lb-s^2/in.

n dynamic modular ratio

n number of runners

n number of walkers

p number of participants in activity

t time, s

v_p peak velocity due to walking, mips

$v_{p,Lim}$ equipment tolerance limit expressed as peak velocity, mips

w uniformly distributed weight per unit length (actual dead and live loads, not design loads) supported by the member, kip/in.

w supported weight per unit area of the panel, psf

w_p unit weight of rhythmic activity with participants distributed over the entire bay, psf

w_t distributed weight supported including dead load and superimposed dead load with occupants and participants distributed over the entire bay, psf

x distance measured along the diagonal between supports, in.

x_E distance from girder to sensitive equipment location, measured parallel to the beam, in.

x_R distance from end of stringer to response location, measured on the diagonal, in.

x_W distance from end of stringer to walker excitation force location, measured on the diagonal, in.

x_W distance from girder to walker location measured parallel to the beam, in.

x_s distance measured along the diagonal between stair supports, in.

y_E distance from the reference beam to sensitive equipment location, measured parallel to the girder, in.

y_W distance from the reference beam to walker location measured parallel to the girder, in.

α dynamic coefficient

α_h dynamic coefficient which causes resonance

α_h dynamic coefficient for the *h*th harmonic

α_i dynamic coefficient (ratio of harmonic force magnitude to bodyweight) for the *i*th harmonic

β modal damping ratio

β viscous damping ratio

β_d damping added by the TMD

Δ midspan deflection of the member relative to its supports due to the supported weight, in.

Δ_c axial shortening of the column or wall due to the weight supported, in.

Δ_f narrowband frequency resolution, 0.125 Hz if $T = 8$ s

Δ_g midspan deflection of girder due to the weight supported by the member, in.

Δ'_g reduced girder deflection for constrained bays, in.

Δ_j midspan deflection of the beam or joist and girder due to the weight supported by the member, in.

γ walking load parameter

ϕ mode shape value

ϕ_E unity normalized mode shape value at the sensitive equipment location

ϕ_R unity normalized mode shape value at response (potentially affected observer) location

ϕ_R mode shape amplitude from Equation 2-12 computed at the response location

ϕ_W unity normalized mode shape value at the excitation (walker) location

ϕ_i phase lag for the *i*th harmonic, rad

$\phi_{i,m}$ *m*th mode mass-normalized shape value

ρ resonant build-up factor

θ stair inclination angle from horizontal, measured with respect to support points, degrees

τ time immediately after a footstep application, s

ν Poisson's ratio = 0.2

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